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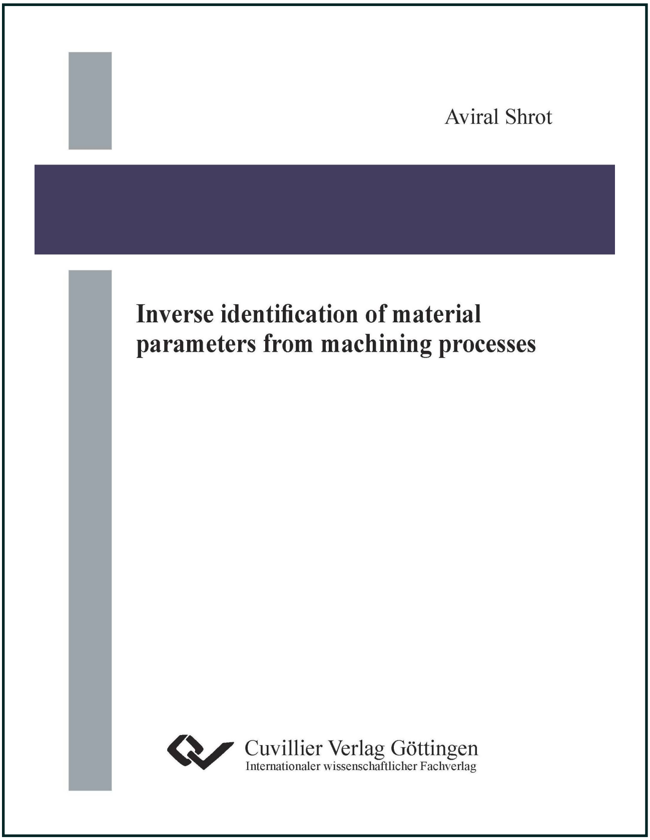
Inverse identification of material parameters from machining processes



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Chapter 1

Introduction

1.1 Motivation

Machining is a widely used manufacturing process in which a workpiece material is progressively given a complex shape by material removal using a tool (Childs *et al.* (2000); Shaw (1984)). The relative motion between a tool and a workpiece is used to remove material in the form of chips. This technique has been used for a wide variety of materials such as metals (Childs *et al.* (2000); Shaw (1984)), non-metals (Alauddin *et al.* (1995); Koch (1964)) and composites (Ferreira *et al.* (1999); Komanduri *et al.* (1991)). Even though this idea of material removal is very simple, the physical understanding of the process is complicated by the complex interactions between the tool and the workpiece. High stresses, strains, strain rates and temperature variations are encountered during the process which have a direct bearing on the energy consumption, tool wear, workpiece surface finish *etc.* A better understanding of the machining process is therefore necessary in order to optimise the process parameters so as to make it more efficient and reduce the wastage of resources.

The mechanics of machining has been studied for more than 100 years (Childs *et al.* (2000)). Early studies of the chip formation process were based on the analytical modelling of the idealised orthogonal chip formation process (see Section 1.5). The analytical modelling was however only useful for analysing very simple cases of chip formation. High speed machining, segmented chip formation, complex machining processes such as milling, drilling, grinding, *etc.* could not be readily modelled using analytical techniques. With the advent of powerful computers and the development of robust numerical algorithms, the simulation of the chip formation process has become an important tool for understanding the mechanics of machining and has also been successfully used for simulating complex cutting processes.

Finite element simulation of the chip formation process is however challenging. The material behaviour under the conditions prevailing during machining is not well-known. Apart from this, the tool and the chip interaction is complex, due to the extreme conditions prevailing near the tool tip. The interaction is also extremely sensitive to the application of external elements such as lubricants and coolants. Both the material properties and the interaction properties are coupled; for instance a change in temperature can lead to a change in the frictional properties. Chemical reactions can also take place between the newly formed surface and the surrounding environment, changing the physical property. All this prevents the modelling of machining processes from being robust.

In the absence of robust cutting models, the manufacturing industry has resorted to creating large machining databases in which machining parameters are recommended for various workpiece material and tool combinations. This approach is not only expensive and time taking, it is also very unreliable. The machining parameters often have to be further tuned by trial and error to achieve the best results. On top of this, new workpiece and tool materials are frequently developed and such databases are rarely available for all possible combinations. Without a robust predictive machining model, a lot of time, money and efforts are thus being spent to determine the optimal machining parameters.

During continuous chip formation at high cutting speeds, in regions near the tool tip, the average strain in the chip can reach up to 200%, strain rates can reach up to the order of 10^5 s^{-1} and a temperature rise of several hundreds of degrees can take place. In Figure 1.1, D_1 schematically represents the domain of state variables - strains, strain rates and temperatures during a machining experiment. For a finite element model of machining, the flow stress as a function of these state variables is required. Phenomenological material models, such as the Johnson-Cook model (see Section 2.2.3), are parametric models which can predict the flow stress as a function these state variables. The main challenge lies in obtaining these parameters at the correct range of state variables. Experimental methods, such as the Split Hopkinson Pressure Bar (SHPB) test, can be used to obtain the flow stress at strains up to 50%, strain rates of the order of 10^4 s^{-1} and temperature rise of hundreds of degrees. In Figure 1.1, D_2 schematically represents the domain of state variables during a typical SHPB test. Using data fitting techniques, the material parameters are obtained from the experimental data. In this case, the material parameters are optimised for predicting the flow stress values in the domain of the state variables reached during the SHPB test. On using these material parameters for high speed machining simulations, extrapolations over several orders of magnitude of strains and strain rates are expected to occur. The material model provides a way for extrapolating the flow stress curves outside the domain at which the parameters are obtained. Since the material parameters are not optimised for the machining conditions, the extrapolated flow stress values in the domain of machining are expected to be inaccurate.

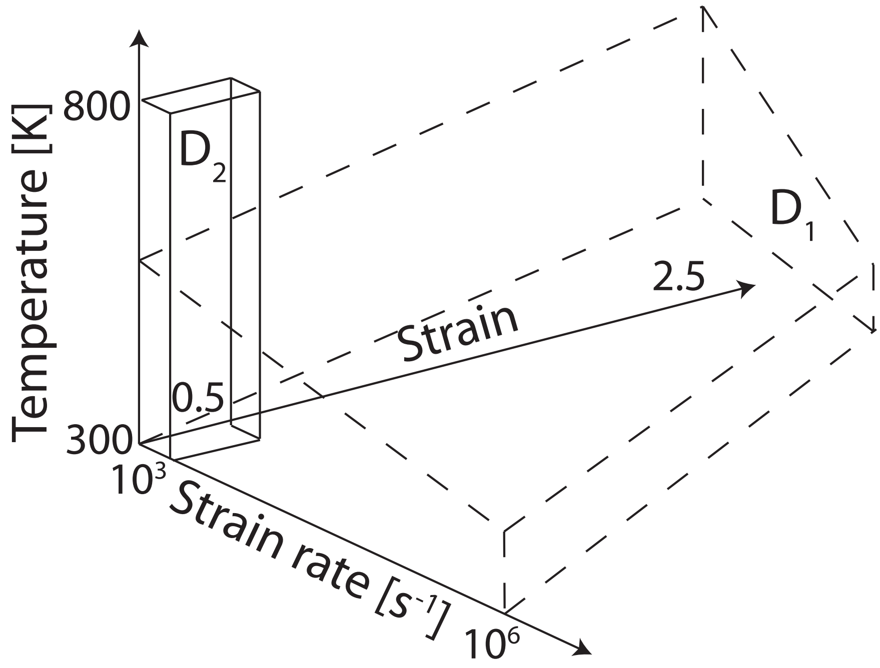


Figure 1.1: The domain D_1 schematically represents the range of strain, strain rate and temperature a typical high speed machining experiment where continuous chips are formed and D_2 schematically represents their range during a typical SHPB test.

To obtain material parameters valid in the domain of machining, parameters can be identified directly from the machining process itself using the inverse identification process. In this method, simulations are conducted with a set of material parameters and the resulting chip shapes and cutting forces are compared to those from the experiments. If the experimental and the simulation results do not match, the material parameters in the simulations are methodically varied until the match occurs. The use of a material model makes the inverse identification process feasible. If the flow stress were to be found as a set of values for different combinations of state variables, inverse identification would have been impossible as a large number of parameters would have to be identified. In a parametric material model like the Johnson-Cook model, the different material parameters can be adjusted in a way that a number of parameter sets give rise to similar flow curves (see Chapter 4). If these parameter sets are optimised over the domain of machining for a given set of cutting condition, similar chips and cutting forces are expected to be formed, as long as the state variables reached are within this domain. If the cutting conditions were changed in such a way that state variables are obtained outside the domain in which the material parameters are determined, extrapolations would occur, making the simulation results unreliable. A robust material parameter set should be optimised for a large range of cutting conditions so that the flow stress can be predicted correctly over a large domain. If the same flow stress curve is correctly represented in a given domain by more than one parameter set, the resulting observables would also be similar. Consequently it should not matter what is the numerical value of the parameter to represent a given stress-strain curve. Even though in this work the inverse identification method is used for identifying parameters for the Johnson-Cook model, this technique can be used for any other material model.

1.2 Overview

The next sections in this chapter give a background to machining and cutting mechanics. In Section 1.3, the orthogonal cutting process is explained along with the associated terminology. The different chip shapes which can be formed during machining and the basics of their characterisation are also discussed. In Section 1.5, the different analytical models of the orthogonal chip formation process are given. Oxley's predictive cutting model (Section 1.5.2) has not only been used for machining predictions, but has also been used for understanding material behaviour at high cutting speeds.

In Chapter 2, the finite element modelling of chip formation has been discussed. A number of modelling issues such as meshing, the choice of material models, the difficulty in modelling material separation, and computational expense are considered here. During the simulation of segmented chip formation, the problem of hourglassing may occur which can destabilise the simulation. Therefore, a detailed study of hourglassing is also discussed in this chapter.

In Chapter 3, the problem of using material parameters obtained from experimental methods is shown with examples. For two alloys of Titanium (Ti-15-3-3-3 and Ti-6246) and a nickel-based superalloy (Alloy 625), Johnson-Cook parameters are obtained from SHPB experiments.

Machining simulations are then conducted using these parameters, and the chip shapes and the cutting forces obtained from simulations are compared to experiments. The limitations of this method are then discussed in detail. The Johnson-Cook model is also modified for simulating chip formation in Alloy 625.

In Chapter 4, the problem of the non-uniqueness of parameter sets arising from the use of the Johnson-Cook model is studied. The Johnson-Cook model is analysed to understand under what conditions the different parameter sets give rise to similar chips and cutting forces. Finally, recommendations are made to choose the cutting conditions during the inverse identification process in order to identify material parameters which are optimised over a large machining domain.

In Chapter 5, the inverse identification method is discussed. The chip shapes and the cutting forces from standard numerical machining experiments and test simulations are matched to inversely identify the parameters. An error function is created which takes into account the chip shape and the cutting force mismatch. The error function is then minmised using different optimisation strategies. After verifying that this method works for simple cases, a number of identification studies are done to understand the effects of optimisation parameters on convergence and how to improve it. Identification studies are conducted for different cutting conditions and different parameter sets to test the robustness of the method. Finally, using the knowledge of the stress-strain curves, a method of solution improvement is proposed which leads to savings in the computational expense.

In this work, the state-of-the-art and the related previous works are discussed in each chapter. The results in Chapter 3 are a part of a collaborative work. Furthermore, parts of this work have been previously published in international conferences and journals. The publications associated with different chapters are listed below:

Chapter 3: Hokka *et al.* (2012a), Hokka *et al.* (2012b), Hokka *et al.* (2012c)
Chapter 4: Shrot & Bäker (2010), Shrot & Bäker (2012b)
Chapter 5: Shrot & Bäker (2011a), Shrot & Bäker (2011c), Shrot & Bäker (2011b), Shrot & Bäker (2011d), Shrot & Bäker (2012a)

1.3 Basics of Machining

The machining processes can be divided into two broad categories (DIN 8589): ones in which the cutting edges are geometrically well defined, such as turning, milling, drilling, shaping *etc.* and ones in which the cutting edges are geometrically undefined, such as grinding, honing, lapping etc. The processes of the first type are often called "cutting processes" and those of the second type are called "abrasive processes" or "grinding processes". The focus of this work is the study of a simplified cutting process with geometrically defined edge called the orthogonal cutting process, in which the cutting edge is perpendicular to the workpiece motion. A schematic diagram of the orthogonal cutting process has been shown in Figure 1.2.

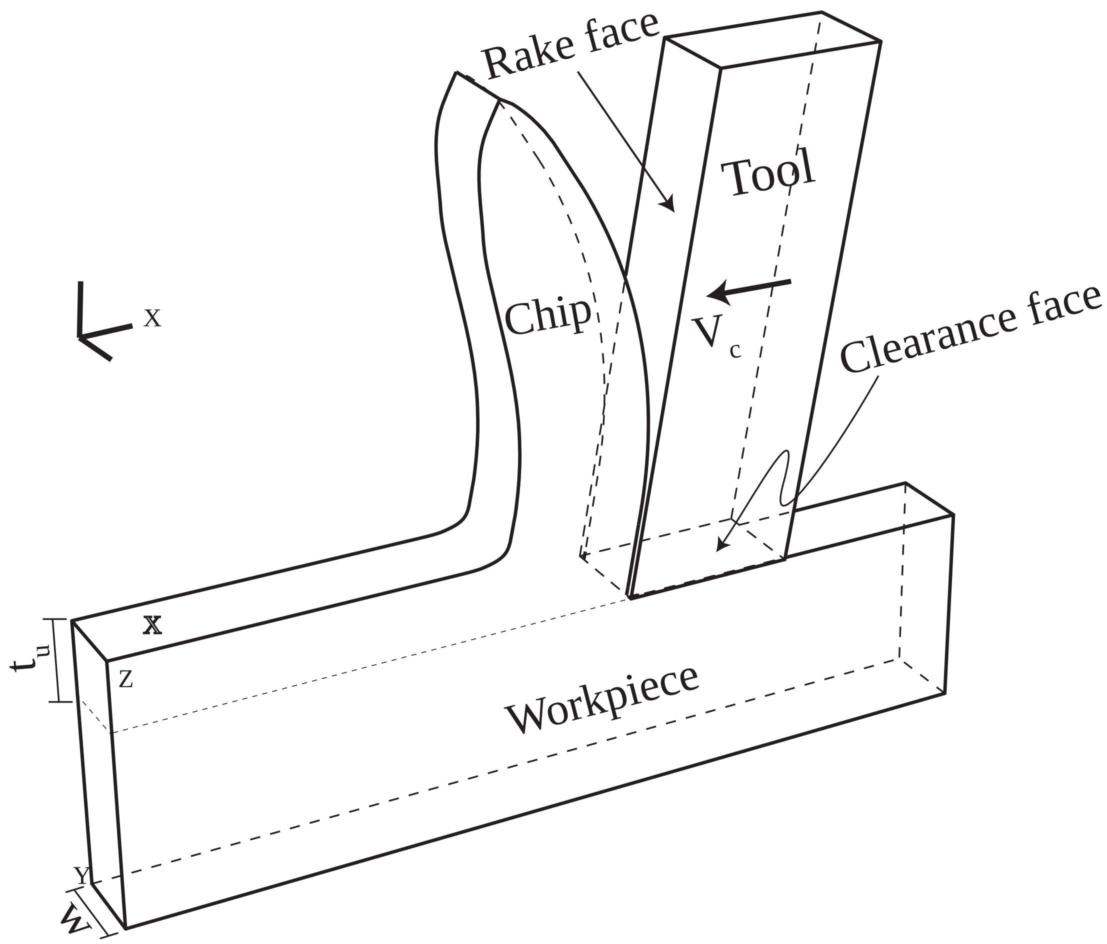


Figure 1.2: A schematic diagram of orthogonal cutting. The cutting edge is perpendicular to the cutting velocity and the chip slides across the rake face.

During this idealised cutting process, the cutting edge is perpendicular to the cutting velocity and the chip slides across the rake face, without

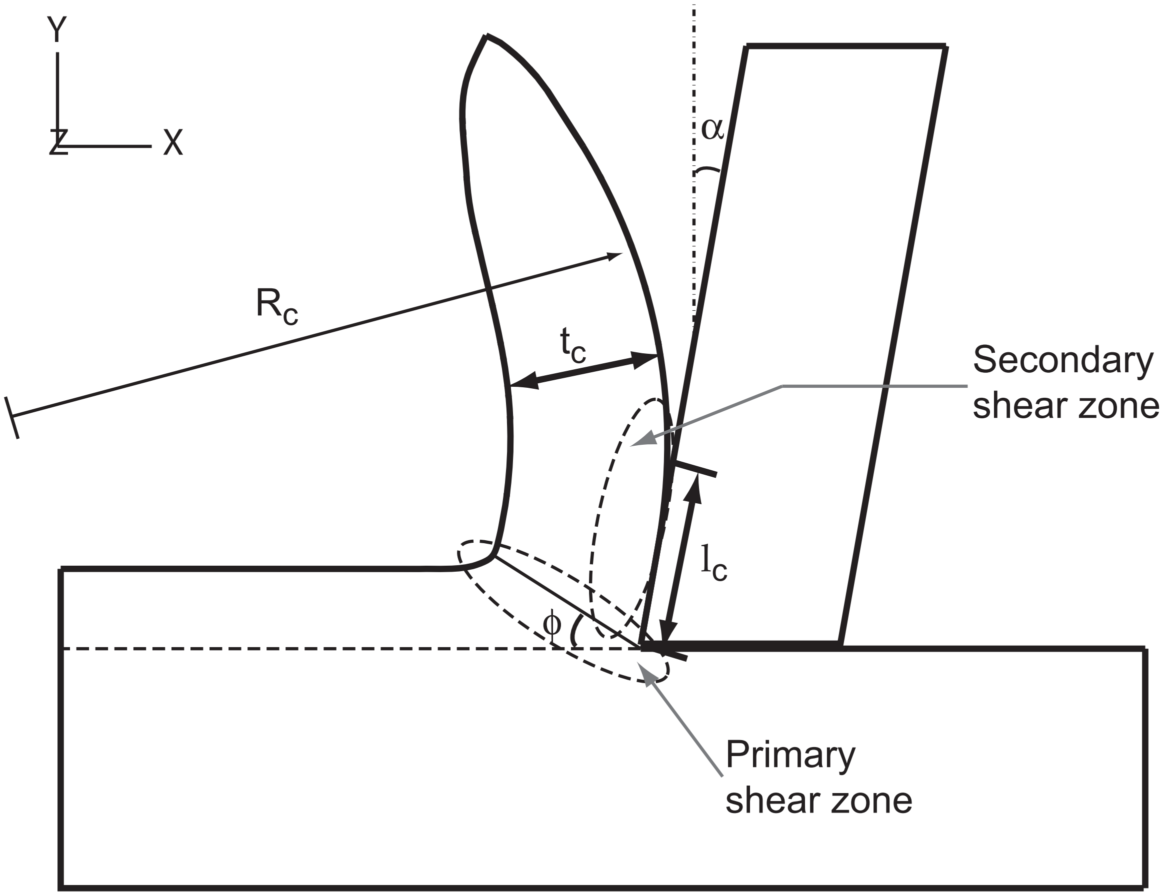


Figure 1.3: A two dimensional representation of orthogonal cutting showing the notable dimensions in a continuous chip.

any curl (Figure 1.2). If the cutting depth is much smaller than the width of cut, then the width of the chip is almost equal to the width of cut. This simplification leads to the assumption of plane strain conditions during the chip formation process: the cutting process can be analysed in two dimensions only, and the physical parameters such as stress, strain, temperature *etc.* are assumed not to vary along the z-direction. In a realistic situation, some increase in the chip width is expected (Lee & Shaffer (1951)), and the stress and temperature distributions in the outer surfaces must be different from those of the interior due to the difference in the heat dissipation. Due to some stochastic instabilities in the material, some chip curl can also occur.

The speed of the tool with respect to the workpiece is called the cutting speed (V_c) and the depth of cut is the same as the uncut chip thickness (t_u). In this case the feed is also the same as the depth of cut since, after each pass, the tool will move down by a distance equal to the depth of cut. The chip has some measurable dimensions (Figure 1.3) such as the chip thickness (t_c), the chip curvature (R_c) and the chip contact length (l_c). The chip thickness is easy to define in a continuous chip (see Section 1.4) where it is more or less constant. However, in non-continuous chips, an average chip thickness can be defined (Cotterell & Byrne (2008)).

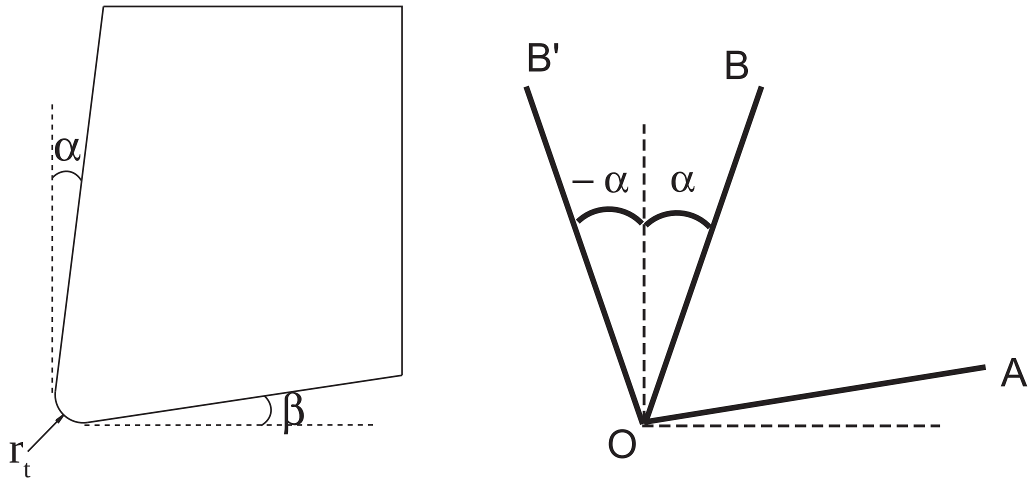


Figure 1.4: a) A schematic representation of the tool showing the rake angle (α), the clearance angle (β) and the tool tip radius (r_t). b) Convention adopted for measuring positive and negative rake angles.

The tool geometry (Figure 1.4a) has a pronounced effect on the chip formation. The rake angle (α) is defined as the angle between the vertical and the rake face. The convention for measuring positive and negative rake angles are shown in Figure 1.4b. Surfaces OB or OB′ represent the rake face of the tool and the surface OA represents the flank face of the tool. Cutting experiments and simulations (Ceretti *et al.* (1999); Günay *et al.* (2004); Lo (2000); Shih (1995); Worthington (1975)) have shown that the rake angle variation affects the observables such as the chip shape and the cutting force profoundly. The clearance angle (β) is defined as the angle between the horizontal and the flank face. The clearance angle has a relatively smaller influence on the cutting mechanics, it is however important for the quality of the machined surface: a positive clearance angle is necessary, so as to allow the tool to clear the machined surface without any interference. This interference can be caused by the spring back of the workpiece or the vibrations of the tool and the workpiece. However, if the clearance angle is too large, the tool tip's strength may be reduced and it will be prone to breakage. It has also been shown that the clearance angle has a bearing upon the flank face wear; an optimum clearance angle of about 6° is recommended so as to balance the flank wear and workpiece surface quality (Moneim *et al.* (1981)). The tool nose radius has also been shown (Chou & Song (2004); Liu *et al.* (2004); Yen *et al.* (2004)) to have a significant influence on the chip shape, cutting forces, surface roughness, residual stresses etc.

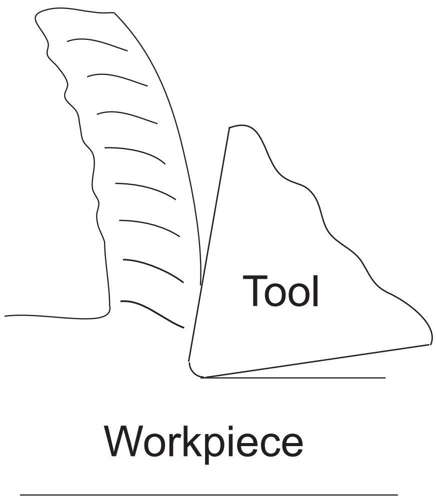
The tool advance which causes the material deformation and overcomes the frictional force between the tool and the workpiece results in a force on the tool. In orthogonal cutting, the force acting on the tool can be resolved into two components, one in the cutting direction which is called the cutting force (F_c) and the second in the vertical direction called the passive or the thrust force (F_t). When comparing cutting force measurements from different experiments, it is reasonable to compare the specific cutting force, *i.e.* the cutting force divided by the product of the uncut chip thickness and the cutting width, which has the units of pressure or energy density.

Large plastic deformations occur during the chip formation process. In early chip formation theories (Ernst & Merchant (1941); Merchant (1945a); Piispanen (1948)), it was assumed that most of the deformation takes place in an infinitesimally thin plane called the shear plane. The angle at which this plane is inclined to the horizontal is called the shear angle (ϕ) (Figure 1.3). It has been later shown (Oxley & Welsh (1963)) that this is an idealisation and that the workpiece material is gradually deformed over a zone called the primary shear zone. The chip and the tool interact to cause further plastic deformation called the secondary shear zone.

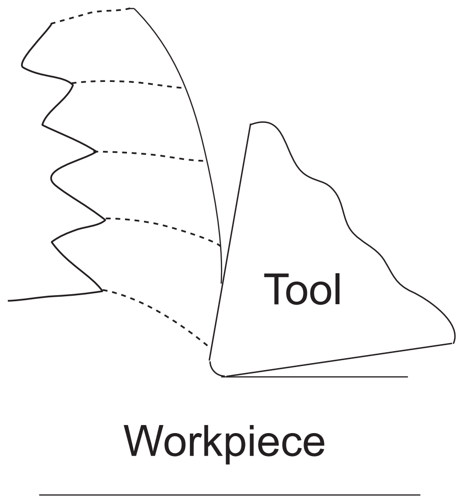
1.4 Chip Morphology

Based on the chip morphology, the chip shapes can be divided into the following broad categories (shown in Figure 1.5):

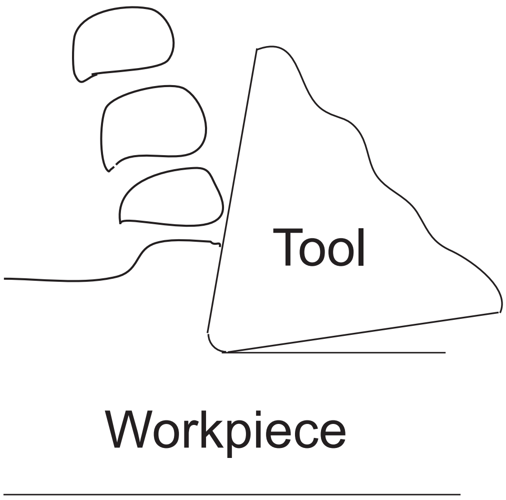
Continuous chip: Continuous chips can be characterised by their practically smooth surface with a constant chip thickness. These chips are formed in a stationary process and the deformation in the chip is more or less uniform. Such chips are often associated with ductile materials. The resulting surface roughness is good as the tool vibration is reduced due to a stable chip formation process. However, this is bad for the automation of the process since long ribbon-like chips can get entangled around the tool holder and the cutting process has to be interrupted to remove the chips from the process zone.



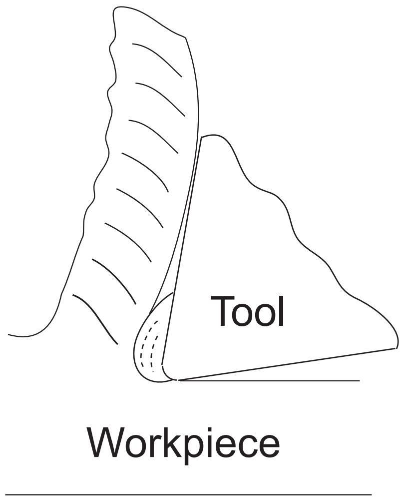
(a) Continuous chip



(b) Segmented chip



(c) Separated chip



(d) Built-up edge (BUE) chip

Figure 1.5: Different chip morphologies

Segmented chip: Segmented chips, also called serrated chips or saw tooth chips are characterised by a series of peaks and valleys in the chip thickness. The region between two valleys, called a segment, is weakly deformed and most of the large deformation concentrates in a narrow region, called the shear band. Such chips are often formed under high cutting speeds and materials with low thermal conductivity are prone to form such chips. The geometrical parameters used to characterise a segmented chip are shown in Figure 1.6. An average chip thickness may be defined for the segmented chips as

$$t_c^{av} = \frac{h_{max} + h_{min}}{2}. \quad (1.1)$$

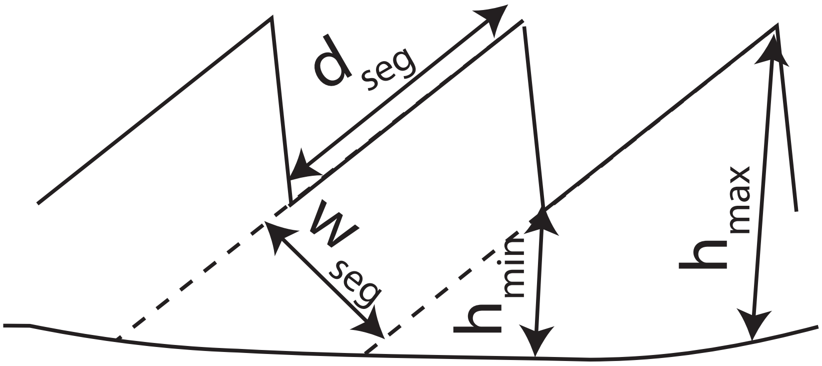


Figure 1.6: Geometrical parameters used for characterising segmented chips. h_{max} is the maximal segment height, h_{min} is the minimal segment height, w_{seg} is the segment width and d_{seg} is the sheared distance.

Average chip compression factor for a segmented chip is given by

$$\lambda_c^{av} = \frac{t_c^{av}}{t_u} . \tag{1.2}$$

The degree of segmentation is defined as

$$g_{seg} = \frac{h_{max} - h_{min}}{h_{max}} . \tag{1.3}$$

The value for g_{seg} is 0 for a continuous chip and 1 for a separated chip.

Separated chip: Separated chips or discontinuous chips are formed when the chip segments get completely separated. The cutting force varies rapidly when the segments break away, leading to tool vibrations and increased surface roughness in the case of ductile materials. However, the machining of such materials can be readily automated since short breaking chips are formed.

Built-up edge chip: Built-up edge (BUE) chips are formed due to the adherence of the workpiece material onto the tool surface, often due to extreme conditions of high temperature and pressure existing near the tool tip. Chemical interaction between the workpiece material and the tool material is also one of the factors which leads to its formation. The built up chips grow in size gradually, accumulating more and more material, until they become unstable and break away. Built up edge chips are detrimental to the surface roughness and also lead to faster tool wear.

1.5 Analytical description of cutting mechanics

In this section, some major results from analytical models of the chip formation process will be discussed in brief. An in-depth analysis of the methods can be found in (Childs *et al.* (2000); Oxley (1989); Shaw (1984)). The most important parameters of interest which can be found by such models are usually the cutting force, average strain, average strain rate and average temperature. Out of these, the cutting force prediction is especially important in industrial settings since it gives the process designer some idea about the input power requirements. The cutting force is also an important criterion to help in tool design and selection, as are parameters such as the rake angle and the clearance angle. Strain, strain rate and temperature predictions are of a greater interest to the material scientist, who can better understand the material behaviour at these conditions and help in the development of new materials and processes (Rösler *et al.* (2005)).

1.5.1 Shear plane model

One of the earliest analytical models was the shear plane model (Ernst & Merchant (1941)) for orthogonal cutting. One of the most important assumptions in this theory was that all the shear deformation took place in a single shear plane. This assumption was based on Ernst's (Ernst (1938)) observations that the continuous chip are formed with most of the deformation taking place in a very thin zone going from the tool to the chip top surface.

Assumptions used in the shear plane model are (also summarised in Shaw (1984)):

1. The material is ideally rigid and perfectly plastic.
2. The tool is perfectly sharp and there is no interaction between the flank face and the workpiece.
3. The shear occurs along a plane called the shear plane
4. The cutting velocity is uniform.
5. The cutting edge is perpendicular to the cutting velocity.
6. The uncut chip thickness is constant.
7. The tool width is greater than the width of cut.

8. A continuous chip forms without a built up edge
9. Plane strain conditions exist.
10. The stresses on the shear plane and the tool are uniform.

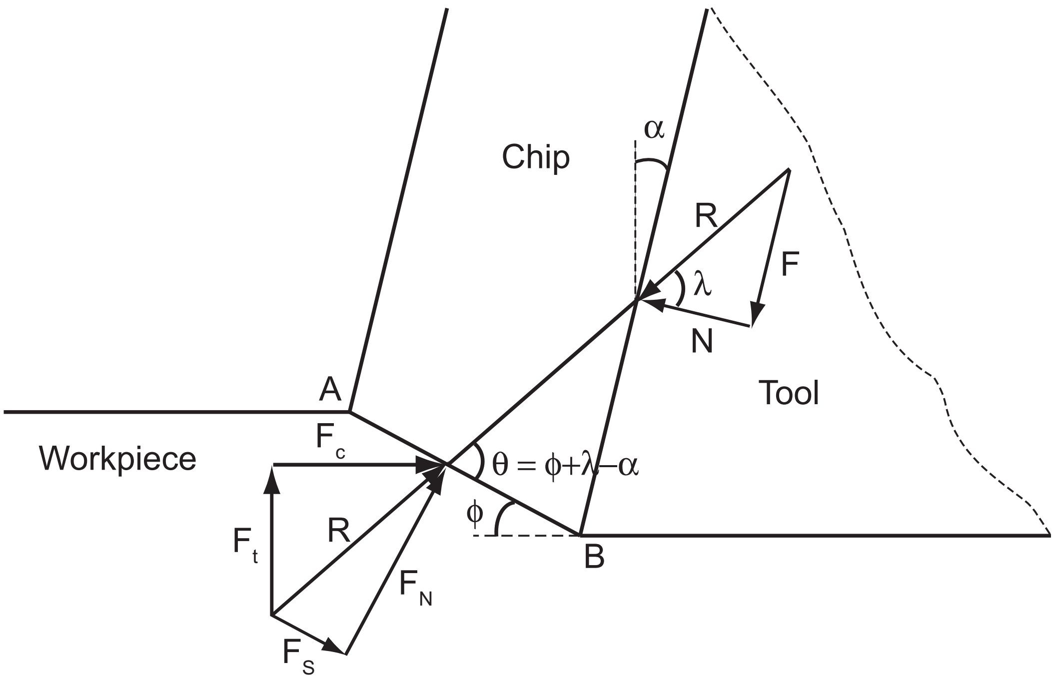


Figure 1.7: The shear plane model along with the associated forces.

Figure 1.7 shows a schematic diagram of the shear plane model. The cutting speed is V_c, the chip speed is v, the uncut chip thickness is t_u, the chip thickness is t_c, the rake angle is α and the shear angle is ϕ. The length of the shear plane AB is l. The chip speed is related to the cutting speed by

$$v = \frac{V_c \sin\phi}{\cos(\phi - \alpha)} . \tag{1.4}$$

The velocities along (v_S) and normal (v_N) to the shear plane can also be related to the cutting speed using

$$v_S = \frac{V_c \cos\alpha}{\cos(\phi - \alpha)} , \tag{1.5}$$

$$v_N = V_c \sin\phi . \tag{1.6}$$

The shear strain undergone by the material across the shear plane is given by

$$\gamma_S = \frac{v_S}{v_N} \tag{1.7}$$

$$= \frac{\cos\alpha}{\sin\phi\cos(\phi - \alpha)} . \tag{1.8}$$

Due to the assumption that all the shear occurs across an infinitely thin shear plane, the strain rate is infinite. Using the constancy of volume, a relation between the chip thickness and the uncut chip thickness can be obtained:

$$V_c t_u = v t_c \,, \tag{1.9}$$

or

$$\frac{t_u}{t_c} = \frac{v}{V_c} \tag{1.10}$$

$$= \frac{\sin\phi}{\cos(\phi - \alpha)} \,. \tag{1.11}$$

The cutting force (F_c) and the thrust force (F_t) can be expressed in terms of R which is the resultant force transmitted across the tool-chip interface and λ which is the mean angle of friction (Figure 1.7):

$$F_c = R\cos(\lambda - \alpha) \,, \tag{1.12}$$

$$F_t = R\sin(\lambda - \alpha) \,. \tag{1.13}$$

The resultant force R can also be expressed in terms of F_S, the shear force along the shear plane as:

$$R = \frac{F_S}{\cos\theta} \tag{1.14}$$

$$= \frac{k_S t_u w}{\sin\phi\cos\theta} \,, \tag{1.15}$$

where θ is the angle made by the resultant R with the shear plane and k_S is the shear flow stress along the shear plane. The resultant force can be resolved along the rake face as:

$$F = R\sin\lambda \,, \tag{1.16}$$

and normal to the rake face as

$$N = R\cos\lambda \,. \tag{1.17}$$

The shear angle (ϕ) is an unknown quantity which is required in calculating the cutting force as well as the parameters which determine the chip geometry. Therefore it is important to have an analytical estimate of the shear angle. By choosing ϕ so that the expenditure of work is minimised (Ernst & Merchant (1941); Merchant (1945a,b)), the shear angle is given by:

$$\phi = \frac{\pi}{4} + \frac{\alpha}{2} - \frac{\lambda}{2} \,. \tag{1.18}$$

Using a similar argument for the minimisation of energy, but for a material whose shear flow stress increased linearly with the increase in the normal stress in the shear plane (Merchant (1945a,b)), the shear angle was predicted to be

$$\phi = \frac{\cot^{-1} s_f}{2} + \frac{\alpha}{2} - \frac{\lambda}{2}, \tag{1.19}$$

where s_f is the slope of the shear flow stress against the normal flow stress relation.

The shear plane theory was the first analytical method to study the chip formation process. However, there were a number of weaknesses in this model. Most significantly, a realistic material behaviour was not considered and the assumption that all deformation took place in a shear plane was an approximation. It has been seen (Oxley (1989)) that the deformation takes place in a zone called the shear zone where the workpiece material gets gradually deformed into the chip. This zone may be thick or thin, based on parameters such as the material plasticity, cutting speed, heat conductivity etc. At higher cutting speeds, where the shear zones are expected to be thinner, there is a greater likelihood of saw tooth chip formation, which is not handled by this model. The dependence of this model on the *a priori* knowledge of the shear angle makes it difficult to make accurate predictions since the shear angle relation is not well known at a wide range of cutting parameters. The minimum work assumption used in the model is also shown to be incorrect by Bäker (2005).

1.5.2 Oxley's predictive machining theory

Oxley and his co-workers have worked extensively in developing an analytical theory for predicting the cutting force and the shear angle during continuous chip formation. The assumption that most of the plastic deformation occurred gradually over a parallel sided shear zone instead of occurring on a single shear plane allowed for the estimation of the strain rates in the shear zone (Oxley & Welsh (1963)). The shear strain rate along AB is given by:

$$\dot{\gamma}_{AB} = C_{ox}\frac{v_S}{l}, \tag{1.20}$$

where C_{ox} is a dimensionless constant and l is the length of AB with

$$l = \frac{t_u}{\sin\phi}. \tag{1.21}$$

A simple material hardening model was considered such that the flow stress

$$\sigma_p = \sigma_1 \varepsilon_p^n \,, \tag{1.22}$$

where σ_1 and n are material constants and ε_p is the plastic strain. It is related to the shear strain using the relation:

$$\varepsilon = \frac{\gamma}{\sqrt{3}} \,, \tag{1.23}$$

and consequently the plastic strain rate is related to the shear strain rate

$$\dot{\varepsilon} = \frac{\dot{\gamma}}{\sqrt{3}} \,. \tag{1.24}$$

The angle θ which the resultant R makes with the shear plane is given by:

$$\tan\theta = 1 + 2\left(\frac{\pi}{4} - \phi\right) - C_{ox} n \,, \tag{1.25}$$

which from geometry is also equal to

$$\theta = \phi + \lambda - \alpha \,. \tag{1.26}$$

The strain at AB is given by:

$$\gamma_{AB} = \frac{1}{2}\frac{\cos\alpha}{\sin\phi\cos(\phi-\alpha)} \,, \tag{1.27}$$

which is half of what is predicted by the shear plane theory. The temperature at AB is given by:

$$T_{AB} = T_w + \eta \Delta T_{SZ} \,, \tag{1.28}$$

where T_w is the initial workpiece temperature, ΔT_{SZ} is the temperature rise in the shear zone and η is a factor $(0 < \eta \leq 1)$ to account for the fact that not all of the plastic deformation occurs at AB. The temperature rise in the shear zone can be calculated by considering the plastic work done.

To calculate the flow stress using the Equation 1.22 at given temperature and plastic strain rate, a velocity modified temperature term is introduced which is used to find the values of σ_1 and n at the given conditions (Oxley (1989)). The velocity modified temperature term is given by:

$$T_{mod} = T\left[1 - \kappa_{ox} \ln \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_{p0}}\right] \,, \tag{1.29}$$

κ_{ox} is a constant and $\dot{\varepsilon_{p0}}$ is the reference plastic strain rate ($\kappa_{ox} = 0.09$ and $\dot{\varepsilon}_{p0} = 1\text{s}^{-1}$ for plain carbon steels in the works of Oxley (Oxley (1989))).

Further analytical equations were developed to estimate the shear zone temperature, temparature at the tool-chip interface, the chip contact length *etc.*, the details of which can be found in Oxley (1989).

1.5.3 Algorithm for machining calculations using Oxley's theory

Oxley's predictive machining theory can be used to calculate the shear angle and the cutting forces given the cutting conditions and the material properties. The cutting force is calculated so that the work is minimised. The assumption of work minimisation during chip formation has later been shown to be incorrect (Bäker (2005)). Oxley's predictive machining theory is however useful as it provides an analytical method for estimating the cutting forces, which is required to find the power requirements for the machining process. This method has also been used by a number of researchers to estimate the flow stress at machining conditions and finding the corresponding material parameters (discussed in Section 5.1). Whereas Oxley used the Power law material model for machining predictions, some researchers extended his theory also for the Johnson and Cook model (Lalwani *et al.* (2009)). Oxley's theory has thus proved to be a useful analytical tool for machining predictions.

One of the major limitations of this method is that segmented chip and separated chip formation cannot be studied with it. Another disadvantage is that only average estimates are available for a number of physical quantities, which is insufficient. For example, chips with completely different shapes can be formed with similar average cutting force values (Bäker (2003a)). In recent times, numerical techniques such as the finite element method are used to study the chip formation process. They provide a better understanding of the physics of the chip formation process. The use of the finite element method for the chip formation process is described in Chapter 2.