



Kai-Simon Goetzmann (Autor)

**The Power of Compromise**

Reference Point Methods and Approximation in Multicriteria  
Optimization

Kai-Simon Goetzmann

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