



Kai-Simon Goetzmann (Autor)

**The Power of Compromise**

*Reference Point Methods and Approximation in Multicriteria Optimization*

Kai-Simon Goetzmann

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in Multicriteria Optimization

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