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# New numerical approaches to model hydraulic fracturing in tight reservoirs with consideration of hydro-mechanical coupling effects

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## New numerical approaches to model hydraulic fracturing in tight reservoirs with consideration of hydro-mechanical coupling effects

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## Content

<b>1</b>	<b>Introduction .....</b>	<b>1</b>
<b>2</b>	<b>Governing equations for individual process in hydraulic fracturing .....</b>	<b>6</b>
2.1	Linear elasticity theory in structure mechanics .....	6
2.1.1	Stress and force equilibrium equation .....	6
2.1.2	Strain and geometric equations.....	10
2.1.3	Constitutive equations .....	13
2.1.4	Virtual work principle .....	14
2.2	Fracture mechanics .....	15
2.2.1	Griffith's theory.....	16
2.2.2	Irwin's theory .....	17
2.2.3	Numerical estimation of the stress intensity factor .....	19
2.2.3.1	Stress intensity factor derived by the calculated displacement and stress field ....	19
2.2.3.1	Stress intensity factor derived by <i>J</i> -integral .....	21
2.2.4	Fracture propagation and orientation criteria .....	24
2.2.4.1	Maximal circumferential stress criterion .....	25
2.2.4.2	Maximal energy release rate criterion .....	25
2.2.4.3	Crack open displacement criterion .....	27
2.3	Fluid mechanics in fracture and pore flow .....	28
2.3.1	Fluid characteristics .....	28
2.3.2	Navier-Stokes equations .....	30
2.3.3	Mass conservation equation.....	31
2.3.4	Equation of state for one phase flow .....	32
2.3.5	Governing equations for fluid flow in a fracture .....	33
2.4	Mass transport by fluid.....	37
2.5	Coupling effects between each process in the hydraulic fracturing .....	38
<b>3</b>	<b>Historical views of the developed models for hydraulic fracturing and proppant transport.....</b>	<b>41</b>
3.1	Models for hydraulic fracture propagation.....	41
3.1.1	Basic models in hydraulic fracturing.....	41
3.1.2	2D PKN model .....	43
3.1.3	2D KGD model.....	45
3.1.4	Cell-based pseudo 3D model.....	48
3.1.5	Lumped pseudo 3D model.....	50
3.1.6	Planar 3D model .....	52
3.2	Models for proppant transport and settling.....	54



3.2.1	Basics in proppant transport .....	54
3.2.2	Proppant transport model developed by Barree and Conway .....	58
3.2.3	Proppant transport model developed by Gadde and Liu .....	60
3.2.3.1	Settling velocity model in the Gadde and Liu's transport model .....	60
3.2.3.2	Relative velocity model in Gadde and Liu's transport model .....	66
3.2.3.3	Formulation of proppant transport in the Gadde and Liu's model .....	68
3.2.4	Proppant transport model developed by Hcu et al. ....	69
<b>4</b>	<b>A new numerical 3D-model for simulation of hydraulic fracturing with consideration of hydro-mechanical coupling effects and proppant transport in tight gas reservoirs .....</b>	<b>71</b>
4.1	Basic ideas from the plasticity theory to model discontinuous displacement field .....	71
4.2	Theoretical background and numerical formulation of the newly developed 3D model for simulation of hydraulic fracturing .....	74
4.2.1	Governing equations for mechanical calculation and the developed constitutive model to describe the mechanical behavior of a fracture with consideration of proppant placement .....	74
4.2.2	Developed flow model in fracture with consideration of proppant and its numerical formulation .....	78
4.2.3	Governing equations for porous flow and their numerical formulation .....	82
4.2.4	Flow interaction between fracture and formation pores .....	83
4.2.5	Fracture propagation .....	84
4.2.6	Process coupling and computing schema .....	85
4.3	Verification of the newly developed 3D-model for simulating the whole hydraulic fracturing process including fracture closure and contact, proppant transport and settling .....	86
4.3.1	Numerical modeling of the dynamical growth of a KGD fracture .....	86
4.3.2	Numerical modeling of the proppant transport in a large slot .....	89
4.3.3	Numerical modeling of fracture closure and contact in a fictive 1D model .....	91
4.4	Applications .....	98
4.4.1	Numerical modeling of a laboratory hydraulic fracturing test .....	98
4.4.2	Numerical modeling of a massive hydraulic fracturing operation in the geothermal project GeneSys in Hannover .....	103
4.4.3	Numerical modeling of a hydraulic fracturing operation in a tight gas reservoir in the North German Basin .....	111
<b>5</b>	<b>A new numerical 2D-model for simulation of hydraulic fracture propagation and arbitrary orientation in tight gas reservoirs .....</b>	<b>118</b>
5.1	Governing equations .....	119
5.2	Numerical formulation and implementation .....	120
5.3	Fracture propagation and orientation .....	127
5.4	Verification: propagation of a KGD fracture under fluid injection .....	127



5.5 Numerical examples .....	129
5.5.1 Hydraulic fracturing in a heterogeneous reservoir .....	129
5.5.2 Hydraulic fracturing in a multi-stage fracture system .....	131
5.5.3 Hydraulic re-fracturing after one year production.....	133
<b>6 Conclusion and outlook.....</b>	<b>137</b>
<b>7 References.....</b>	<b>141</b>
<b>Appendix I: .....</b>	<b>146</b>
<b>List of table .....</b>	<b>151</b>
<b>List of figure.....</b>	<b>152</b>
<b>Nomenclature.....</b>	<b>157</b>