

Contents

Vorwort	i
Abstract	iii
Zusammenfassung	v
1 Introduction	1
1.1 Governing Equations	3
1.1.1 Velocity Field	3
1.1.2 Scalar Field	4
1.1.3 Characteristic Numbers	5
1.2 A Statistical Description of Turbulence	5
1.2.1 Random Variables	6
1.2.2 Statistical Symmetries	6
2 Direct Numerical Simulation of Turbulent Flows	9
2.1 Pseudo-Spectral Methods	9
2.1.1 Velocity Field	10
2.1.2 Scalar Field	11
2.1.3 Dealiasing	11
2.1.4 Temporal advancement	12
2.2 Code Design and Parallelization	13
3 Statistical Properties of Turbulence Obtained by DNS	17
3.1 Introduction	17
3.2 Simulation Parameters	17
3.2.1 Characteristic Quantities	18
3.2.2 Resolution Condition	23
3.2.3 Structure of Turbulent Fields Obtained by Direct Numerical Simulation	24
3.3 Phenomenology of Small-Scale Turbulent Mixing	24
3.3.1 Velocity and Scalar Spectra	30
3.3.2 Probability Density Functions	34
3.3.3 Energy and Scalar Dissipation Anomaly	35
3.3.4 Basic Equations	38



Contents

3.3.5	Structure Function Analysis	42
4	Dynamics of Turbulent Mixing	45
4.1	Introduction	45
4.2	The Generalized Yaglom Equation	46
4.2.1	Governing Equations	46
4.2.2	Derivation of the Generalized Scale-by-scale Energy Budget Equation	46
4.2.3	Direct Numerical Simulation	53
4.2.4	Evaluation of the Generalized Scale-by-scale Energy Budget Equation	54
4.3	Dependence of the Scale-by-scale Energy Budget on the Schmidt Number	56
4.4	Scale-by-scale Energy Budget Equation for Coarse-grained Fields and LES	58
4.4.1	Derivation of the Equation	58
4.4.2	Large-Eddy Simulation	62
4.4.3	A-priori Analysis of the LES Budget	65
4.4.4	A-posteriori Anlysis of the LES Budget and its Single Terms	66
4.4.5	Schmidt Number Analysis	68
5	Decomposition of Turbulent Fields by “Turbulent Line Segments”	73
5.1	Introduction	73
5.2	Numerical Method	75
5.3	Phenomenology of Turbulent Line Segments	76
5.4	A PDF Equation for the Mean Length	81
5.5	The Scaling of the Mean Length	84
5.5.1	Derivation Based on the PDF-Equation for the Mean Length	86
5.5.2	Derivation Based on Statistical Properties	87
5.5.3	The Mean Length of Turbulent Line Segments Based on the Velocity and the Scalar Gradient	90
5.6	Compressive and Extensive Strain	91
5.7	Statistical Description of Turbulent Line Segments	92
5.7.1	The Distribution of $\Delta\phi$ and ℓ	92
5.7.2	Structure Functions and Conditional Moments	95
5.7.3	The Distribution of g and ℓ	99
5.7.4	Scaling of the Normalization Quantities with the Reynolds Number	102
5.7.5	Cliff-Ramp Structures	105



6 Regular and Anomalous Scaling of Two-Point Statistics and Gradients	113
6.1 Introduction	113
6.2 Structure Functions and Conditional Moments	114
6.2.1 Reynolds Number Dependence and Scaling Laws	120
6.2.2 A Scale-by-scale Budget Equation for Second and Fourth Order Moments of the Scalar Increment	123
6.2.3 Closure of the Turbulent Transport Term	130
6.3 Statistics of Local and Mean Gradients	131
6.3.1 Flatness of Turbulent Line Segments	131
6.3.2 Reynolds Number Dependence and Scaling Laws	132
6.3.3 Kolmogorov's Refined Similarity Hypothesis	135
6.4 Reconstruction of Fine-Scale Statistics	137
6.4.1 Relation between the moments of ϕ_x and g	138
6.4.2 Reconstruction of the Marginal Gradient PDF	140
7 Summary and Conclusion	147
A Normalization of the Scalar Gradient PDF	149
B Skewness of the Longitudinal Velocity Gradients	150
C Normalized Moments of the Energy Dissipation	152
D Reynolds Number Dependence of the Scalar Flux Spectrum	153
References	155