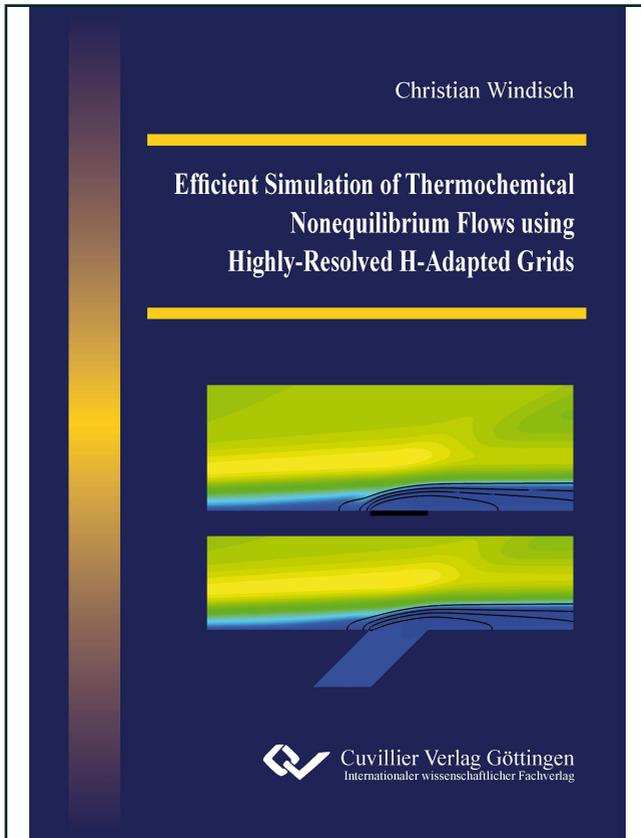




Christian Windisch (Autor)  
**Efficient Simulation of Thermochemical  
Nonequilibrium Flows using Highly-Resolved H-  
Adapted Grids**



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