1 Introduction

1.1 Motivation and objectives

The flows in the interior of planets, in the deep ocean and in the atmosphere are complex and influenced by a large number of physical and chemical factors. To understand the influencing factors it is useful to constrain the investigation of the system to certain partial aspects. Due to the complexity of the problem, the research then concentrates just on the fundamental processes, rather than the exact replication of a particular phenomena. A shell-model simplifies and scales the flow phenomena in the interior of a planet between an inner core and an outer fixed crust. In our experimental study the shell globally rotates at angular velocity Ω. A further periodic oscillation with angular velocity $0 \leq \omega \leq 2\Omega$, a so-called longitudinal libration, is added to the inner sphere's rotation. The primary response is an inertial wave spawned at the critical latitudes on the inner sphere, and propagating throughout the shell along inclined characteristics. For sufficiently large libration amplitudes the higher harmonics also become important. Those harmonics whose frequencies are still less than 2Ω behave as inertial waves themselves, propagating along their own characteristics. The steady component of the flow consists of a prograde zonal jet on the cylinder tangent to the inner sphere and parallel to the axis of rotation, which increases with decreasing Ekman number. The jet becomes unstable for larger forcing amplitudes as can be deduced from Particle Image Velocimetry (PIV) observations. Finally, a wave attractor is experimentally detected in the spherical shell as the pattern of largest variance. These findings are reproduced in a 2D numerical investigation of the flow, and certain aspects can be studied numerically in greater detail. One aspect is the scaling of the width of the inertial shear layers and the width of the steady jet. Another is the partitioning of the kinetic energy between the forced wave, its harmonics, and the mean flow. Finally, the numerical simulations allow for an investigation of instabilities, too local to be found experimentally. For strong libration amplitudes the boundary layer on the inner sphere becomes unstable, triggering localised Görtler vortices during the prograde phase of the forcing. This instability is important for the transition to turbulence of the spherical shell flow.

We study the aspects of flow structures in the rotating concentric spherical shell experiment with main focus to the inertial waves. We visualize the waves and the wave attractors in the experiment with quantitative measurement methods as PIV. Here the research concentrates on the ability to locate the waves in the flow-field. Moreover, we confirm already existing numerical results. The experiment provides the application of the results to more recent numerical models, adjusted to the laboratory experiment.

In addition, the comparison between of experiment and the numerical simulations provides the opportunity to verify the numerical code.

The second main objective in this thesis are the Laser Doppler Velocimetry measurements (LDV) on wave attractors in the fluid-stratified tank experiment. Those wave phenomena have been experimentally studied for about 15 years in a rotating box and in rotating cylindrical gap, nevertheless, they are still not sufficiently explored. Laboratory experiments and numerical simulations will help to answer open questions, e.g. on the scaling of internal shear layers and particle motion in the layers. Here we do a first attempt to detect and measure these wave attractors via the LDV technique. We investigate the velocity profile vertical to the attractor.

1.2 This Thesis

The thesis reports on two different wave motion aspects. On the one hand we focus on the occurrences and measurements on wave attractors in a fluid-stratified tank experiment with one sloping wall. On the other hand we investigate waves, their harmonics and attractor in a rotating spherical shell experiment filled with homogenous fluid and compare this with numerical simulations. The first chapter will give an outline to various waves and their behavior in enclosed geometries. Here, we want to review the relevant literature to inertial and gravity waves, both theoretical and experimental. In chapter 2 we describe the fundamental theory for rotating homogeneous flows. Chapter 3 introduces at first the basics of stratified non-rotating fluids. Next we give a description of the tank experiment and show the measured wave attractors with Synthetic Schlieren technique and LDV. We also show a wave attractor in the spherical shell by applying the Empirical Orthogonal Function (EOF) analysis. Chapter 4 will at first describe the spherical shell apparatus and the research methods Laser Sheet illumination (LSI) and PIV. It will put the results of the observed waves in the context of the experimental data and numerical simulations with respect to linear solutions, higher harmonics and instabilities. Finally, chapter 5 will summarize this thesis and discuss some speculative thoughts that might be followed in future works.

1.3 Waves in fluids

Wave phenomena are an important topic in modern and classical physics. Waves propagating disturbances in gas and in liquid media. When a stone is thrown into water a circular wave develops and this is an example of a propagating wave. However, waves also occur by permanent excitation, such as the formation of surface water waves due to strong wind. Waves transport momentum and deliver it locally. Hence, wavedriven mean flows occur. Waves depend on space and time and are characterized by a periodic motion with a propagating wave energy. Particles in a fluid are deflected from their rest position and are forced back to the initial position and overshoot. An oscillation is the result of this repeated process.

Waves differ in deflection and propagation direction and the position of these two axis. Hence they are transversal or longitudinal. Basically, there are three types of waves, which are defined based on their restoring forces. In geophysical flows, like surface water waves, gravity is the determinative restoring force, for sound waves, the pressure gradient and for inertial waves (or gyroscopic waves) and Rossby waves the Coriolis force. Furthermore, when Coriolis and gravity force act together, they are called internal inertia-gravity waves. Inertial and gravity waves have a characteristic 'stratification' necessary for their existence. Tab. 1.1 compiles the three wave types in terms of oscillation plane, restoring force and stratification.

Table 1.1: The three types of waves in comparison among one another.

type of waves	oscillation plane	restoring force	stratification
sound waves	longitudinal	pressure gradient	-
gravity waves (external/internal)	transverse	buoyancy/ gravity	density
inertial waves/ Rossby waves	transverse	inertia/ Coriolis force	angular momentum

In rotating systems, waves play an essential role for the circulation on a planetary scale, like in the atmosphere, in the oceans or in planets with a variety of flow structures. Therefore, flows in rotating, isothermal and homogeneous fluids in spherical shells are interesting concerning their geophysical applications. Indeed, the whole spectrum of rotational aspects and complexity, like wave motion, non-linear interactions, internal boundary layers or different geometries are involved and they are the subject of many numerical and experimental investigations.

Internal waves, in particular the inertial waves and their characteristic reflections are the focus of this thesis. As mentioned before, there are two types of internal waves, which are important for geophysical motions, the gravity waves and the inertial waves. Both have their maximum amplitude inside the fluid. They occur in density stratified fluids, in rotating fluids and in combinations of both cases. The presented gap experiment used in this work allows to investigate inertial waves in spherical shell geometry. It provides the detection of waves inside the fluid regarding both, experimental and numerical investigations. Since there are sparse experimental studies of this phenomenon in a spherical geometry, it shall contribute to the understanding of excitation and formation of internal waves for this particular case.

1.4 Gravity waves

Gravity waves occur as external and internal wave motion. The oscillating plane is perpendicular to the phase speed, the wave is transversal. In both wave motions, the gravity and buoyancy respectively in a stable stratified fluid have the main influence in the generation of such waves. Thereby in the stable case, the fluid with higher density is below the one with lower density. External gravity waves, for example surface waves between water and air, have a high density contrast. One formation mechanism of surface waves can be the Kelvin-Helmoltz instability, where small disturbances in the shear layer of two fluids with different velocities can grow. In contrast, internal gravity waves are inside a density stratified fluid that cannot be mixed easily. An example is the non-stirring of water and oil. Usually, internal waves have larger amplitudes, smaller phase velocities and longer periods than surface gravity waves at the same forcing due to their smaller 'density contrast'.

When a fluid particle is deflected in the vertical plane, an oscillation occurs. The wave propagates along the interface between the two fluids. The free surface of the upper layer moves marginally, because the amplitude decreases beyond the interface. Hence, gravity waves have their maximum amplitude inside the system.

Let us now focus on the continuously stratified case. The deflected particles in the stable stratified fluid oscillate vertically with the so called Brunt-Väisälä-frequency (N) around their rest position. It depends on the gravity g, the density gradient of the stratification ϱ in the z direction against the gravity and reads

$$N^2 = -\frac{g}{\varrho_0}\frac{d\varrho}{dz}, \tag{1.1}$$

where ϱ_0 is the constant mean density. For stable stratification in an incompressible fluid, the 2D dispersion relation reads

$$\omega^2 = N^2\frac{k_x^2}{\hat{k}^2} = N^2\frac{k_x^2}{k_x^2+k_z^2} \quad \text{and in polar coordinate notation} \quad \omega^2 = N^2 cos^2\,\theta \tag{1.2}$$

and describes the relation between the frequency ω and the wave number $\hat{k} = k_x^2 + k_z^2$. The θ denotes the angle between the horizontal and the wave vector $\vec{k} = (k_x, k_z)$. It is important to note, that for external gravity waves usually $\omega^2 > N^2$, whereas for internal gravity waves $\omega^2 < N^2$ holds.

Despite with no sharp density changes gravity waves propagate obliquely through the fluid instead of horizontally along the interface (Manders, 2003) like the thermocline in the ocean (Pichler, 1997). They have the character of interfacial waves since they occur at the thermocline that separates the low density surface water from the deep ocean. Internal waves can propagate over a long distance and can be observed hundreds of kilometers away from their disturbance source. More examples for internal gravity waves in nature are the so called lee waves. They act in the atmosphere on the leeward side of mountains and propagate vertically.

1.5 Inertial waves

The rotation in inviscid, homogeneous and isothermal fluids causes a stable and continuous radial angular momentum stratification in the system. Specified disturbances generate waves similar to internal gravity waves. This analogy between rotation and stratification is discussed by many authors, e.g. Veronis (1970). In contrast to gravity waves, inertial waves are three-dimensional with deflected oscillating particles. A particle deflected from its initial position has a different velocity as solid-body rotation. Due to the conservation of angular momentum, the azimuthal velocity at the new particle's position might be lower or higher than the velocity to the rotating system (depending on the displacement). For this new velocity, the centrifugal force at the deflected position does not balance the pressure gradient. The particle experiences a restoring force, namely for rotating systems the Coriolis force.

On a horizontal plane rotating with Ω the horizontally displaced particle oscillates anticyclonically along circles in the opposite to the background rotation with frequency 2Ω. In contrast, in 3D the resulting wave is called inertial wave (Greenspan, 1990; Cushman-Roisin, 1994; Pedlosky, 2003) or gyroscopic wave (LeBlond and Mysak, 1978) and it can propagate through the fluid with frequency $0 < \omega < 2\Omega$.

The distinctive behavior is in its anisotropic dispersion, the energy propagation is normal to the phase velocity and causes specific reflections on solid boundaries (Phillips, 1963; Lighthill, 1978). The wave propagates with an angle $\theta = cos^{-1}(\omega/2\Omega)$ to the rotation axes, whereas the energy propagates normal to the wave vector $\vec{k}$. Fig. 1.1 a) shows the velocity $\vec{u}$ and the acceleration $d\vec{u}/dt$ in an oblique plane, perpendicular to $\vec{k}$ with a pressure gradient parallel to $\vec{k}$. The anisotropy dispersion depends on the frequency of the disturbance. The dispersion relation, the relation between the phase velocity and the wave length, is written as

$$\omega = \pm\frac{2\vec{\Omega}\cdot\vec{k}}{|\vec{k}|} = \pm 2\frac{\Omega_x k_x + \Omega_y k_y + \Omega_z k_z}{(k_x^2 + k_y^2 + k_z^2)^{1/2}} \quad \text{and in polar coordinate notation} \quad \omega = \pm 2\Omega\, cos\,\theta. \tag{1.3}$$

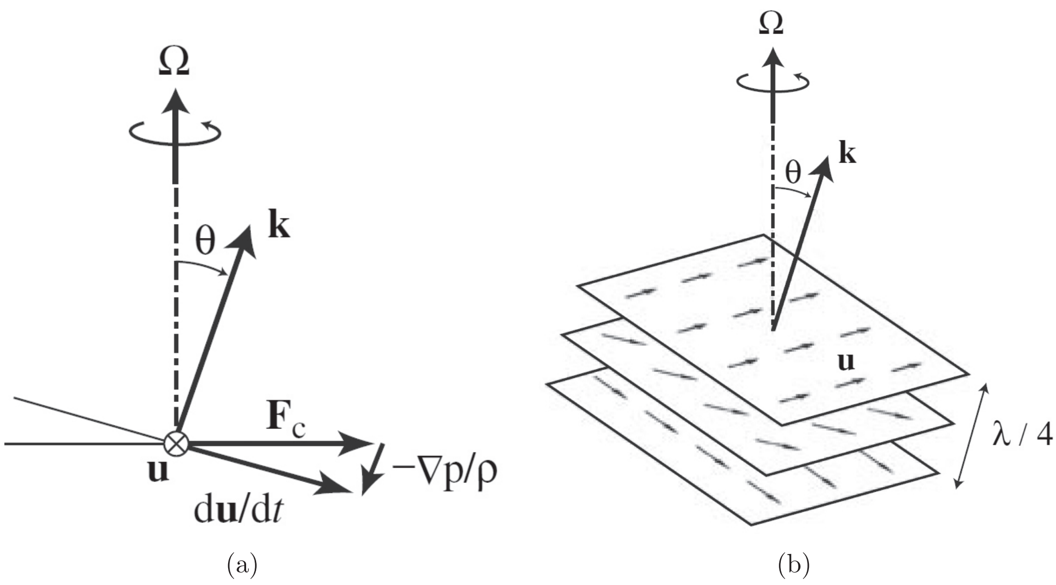
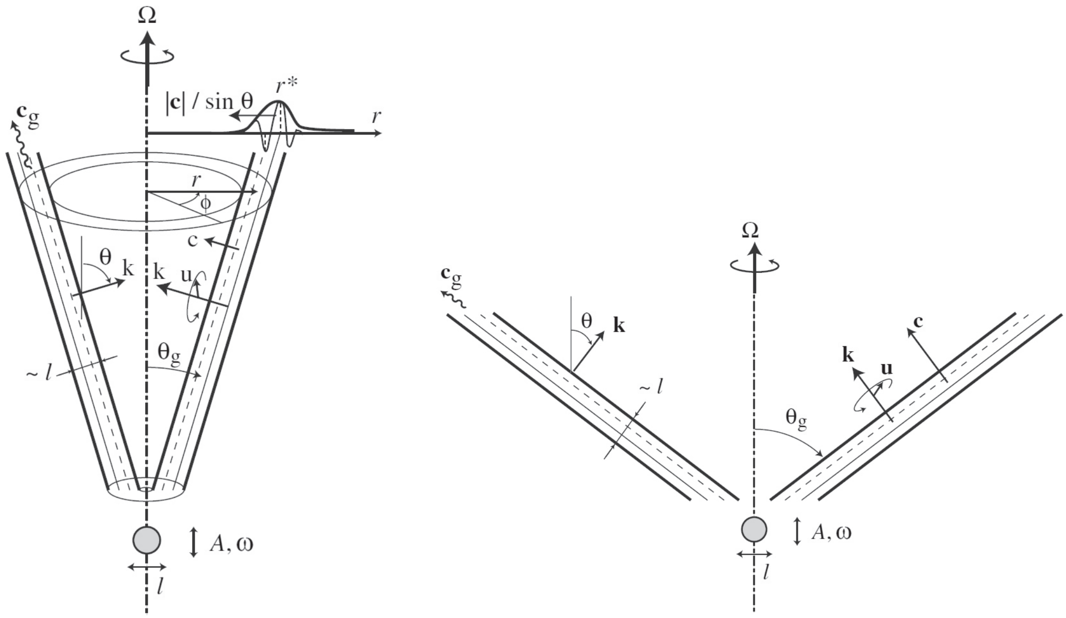


Figure 1.1: (a) 'Coriolis force and pressure gradient experienced by a fluid particle in circular translation with pulsation σ in a frame rotating at angular velocity Ω about the vertical axis. The velocity u and the acceleration du/dt lie in an oblique plane, normal to the wavevector k making angle $\theta = \cos^{-1}(\omega/2\Omega)$ to the rotation axis (here u is chosen normal to Ω). (b) Three planes of constant phase $0, \pi/4, \pi/2$ (corresponding to 1/4 wavelength), normal to k, emphasizing the circularly polarized transverse wave and the shearing motion between planes. The phase velocity is along k, showing that the circular translation is anticyclonic.' Figure and caption taken from Messio et al. (2008).

Note, here the angle θ has a different meaning as in Eq. (1.2). But the dispersion relation of (internal) waves in stratified fluids and inertial waves in rotating fluids is of the same form. Therewith, the properties of waves in a stratified fluid are comparable to the inertial waves in a homogeneous fluid.

Due to the Coriolis force and the pressure gradient, the particles move in circles around the $\vec{k}$ vector with frequency ω. Fig. 1.1 b) illustrates, for constant phase, the circularly polarized traveling wave and the motion of shear between the planes with an anticyclonic phase velocity (Phillips, 1963; Messio et al., 2008). Note, due to $\Omega_x = \Omega_y = 0$ in the illustration of Fig. 1.1, following formula applies $\omega = \pm 2 \frac{\Omega_z k_z}{(k_x^2 + k_y^2 + k_z^2)^{1/2}}$. More precisely, Fig. 1.2 a) and b) show the wave propagating in a slowly oscillating wavepacket as a double cone with angle θ_g with respect to the rotation axes (Messio et al., 2008). In the interior of the cone layer the fluid is oscillating, whereas it is at rest outside. At this, the size of the source, here marked as l, determines the wavelength



(a) $\omega << 2\Omega$ (b) $\omega \sim 2\Omega$

Figure 1.2: Sketch of the cones of inertial waves with increasing frequency ω. Both are forced by oscillating disturbance of size l a and show the upper half cone. Trough and crest in the conical wave packet are illustrated as solid and dashed lines. a) A small oscillation much lower than 2Ω b) Stronger forcing nearly up to angular velocity 2Ω. (Messio et al., 2008)

and the thickness of the cone in the absence of the viscosity. With increasing ω the cone opens up (1.2 b) with changing the direction of the group velocity c_g (Pedlosky, 2003). Hence, for $w \sim 2\Omega$ the wavepacket propagates perpendicular to the rotation axis and the group velocity drops to zero. A stationary planar wavepacket results and the particles circulate horizontal at the frequency 2Ω.

Inertial waves can be distinguished as global (contained) or local waves (short wavelength approximation), depending on how large the wavelength is compared to the container size (Messio et al., 2008). They are important for natural flow motions in the oceans (van Haren and Millot, 2004), the atmosphere (Pedlosky, 2003), liquid cores of planets (Aldridge and Lumb, 1987) or in rotating stars, where the density stratification is weak. In the Earth's atmosphere, inertial waves occur in the form of inertia-gravity waves. Inertial waves are induced by an oscillating homogeneous forcing, like for the ocean tides, or by the temporary variation of the background rotation vector, as for the precessing Earth core (Aldridge and Lumb, 1987). These authors claim that inertial waves are detected in the liquid outer core after large, deep earthquakes. These waves propagate through the fluid as plane waves along rays and show properties of optical waves (Lighthill, 1978; Phillips, 1963). Though, experiments in a finite container filled with inviscid fluid, the provided local waves show also a global behavior due to pattern formation of interference between multiple reflected waves (Messio et al., 2008).

Inertial waves also play a role in the magnetic field of the Earth, since the outer core consists of electromagnetic fluid (Malkus, 1968). The characteristics of the flow motions are also interesting for industrial applications, like in the spacecraft. The space vehicle typically spins around one of its axes during their flight. A fluid-filled tank stabilizes the rotation. Inertial waves may not allow the vehicle to become unstable (Manasseh, 1993).

In 1880, Lord Kelvin (Thomson, 1880) established the investigation of inertial modes, when he considered the vibration of a cylindrical columnar vortex at specific frequencies. Thereafter Bryan (1889) described the analytical solution for inertial waves in a spheroid. The first experimental investigations on inertial waves and their characteristic frequencies, depending on geometries of the systems, have been in cylindrical apparatuses by Oser (1958), Fultz (1959), McEwan (1970), Ito et al. (1984), Manasseh (1996) and Duguet (2006). Recent experimental studies on inertial waves for cylindrical geometry are given in Messio et al. (2008). Investigations for the spherical geometry are given in Aldridge and Toomre (1969) for the full sphere and in Aldridge (1972) for the spherical shell. A steady precession spheroid is investigated by Malkus (1968) and a cone geometry by Beardsley (1970). Features related to wave propagation are detected by Maas (2001) and Manders and Maas (2003) in a rectangular basin with one sloping boundary.

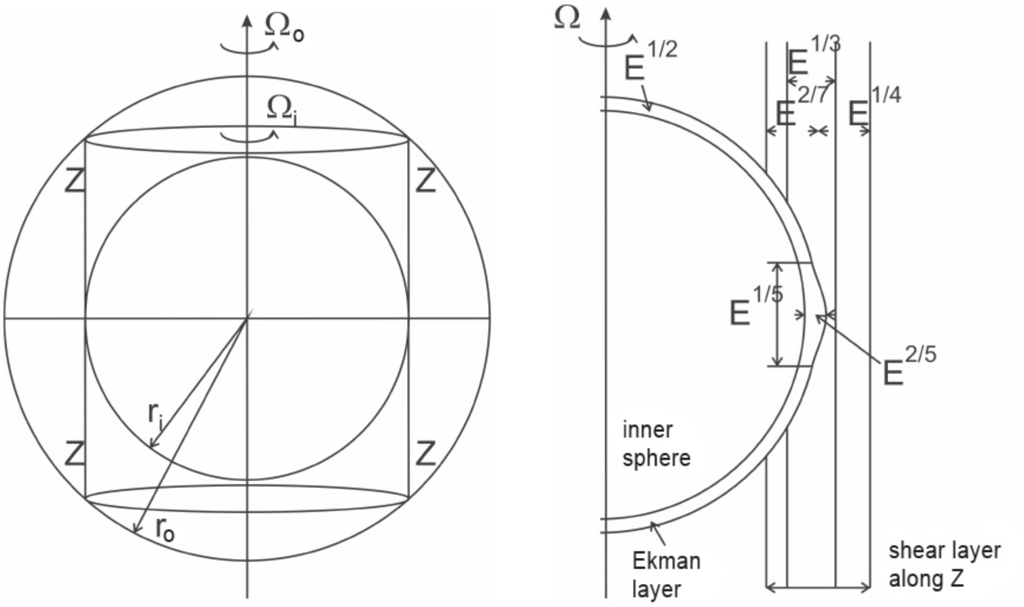
Additionally, recent investigations on inertial waves excitation underline the necessity of further research. Cortet et al. (2010) force inertial waves by a thin oscillating cylinder in a rotating box that generates two-dimensional cross-shaped wave beams and they discuss the viscous spreading of these wave beams. These results are compatible with the solutions of Thomas and Stevenson (1972) for internal waves. Rapidly towing grid-generated turbulence in a rotating tank is investigated by Lamriben et al. (2011). They show a good agreement with the numerical simulations by Maas (2003). The attention of Matsui et al. (2011) is an electrically conducted fluid in a spherical shell under the influence of waves and turbulence on the large-scale flow. Triana (2011) and Zimmerman et al. (2011) worked with this spherical annulus, too. Triana (2011) is interested in the water-filled shell with precessionally forced flow and spin-over inertial modes at differentially rotation of the inner sphere. In contrast, Zimmerman et al. (2011) work with molten sodium metal at low Ekman numbers to measure the hydrodynamic turbulence and the turbulent scaling of the torque on the inner spherical shell. Bordes et al. (2012) show measurements of inertial waves by using oscillating stacked plates in a rotating water tank. They show the excitation of two secondary plane waves, since the primary plane wave is subjected to a subharmonic instability.

In addition to the limited experimental investigations, the numerical studies provide a wide range of parameters. In 1889, Bryan (1889) solved the equations for the inertial modes of standing waves in a sphere. In contrast, Stewartson and Rickard (1969) are interested in the pathological nature of contained inertial modes in a spherical shell. Regarding to this work, Tilgner (1999) excites different inertia modes with varying frequencies in his numerical model. He shows the geometrical influence and importance of the fluid behavior and attractors. By decreasing the radius of the inner sphere, the number of attractors decreases and vanish for the full sphere. Later he shows that a zonal flow can be excited by the interaction of inertial waves (Tilgner, 2007). Numerical simulations of inertial waves in a spherical shell at low Ekman number were done by Simkanin et al. (2010). They show the nonlinear interaction in dependence of the Ekman number for global flows.

Stewartson layer

In rotating, oscillating and librating shells, shear layers, Ekman layer and Stewartson layer occur. They exists in the interior of the fluid or near the boundaries. For spherical shells the Stewartson layer describes the cylindrical shear layer tangential along the inner sphere in a differential rotating system. There the inner sphere rotates with higher velocity than the outer sphere. This tangent cylinder is parallel and axisymmetric to the rotation axes and is separated into three layers with different widths, see Fig. 1.3 b). The higher the rotation rate, the lower is the Ekman number and the thinner is the Stewartson layer.

An overview of different studies of the Stewartson layer in differential rotating systems is given by Hollerbach (2003). The inviscid case is given in the work of Kerswell (1995) and Hollerbach and Kerswell (1995), who study the conical shear layers arising from the critical latitudes. They show further, that internal shear layer are tangential to the boundary layers at the critical latitude. The authors determine the scaling in the interior of the fluid. Also Stewartson (1966), Hollerbach (1994), Hollerbach (2003), Hollerbach et al. (2004) and Fotheringham and Hollerbach (1998) work on the scaling and proof some results. Detailed solutions that include viscosity are solved by Rieutord and Valdettaro (1997), Rieutord (2001) and Rieutord and Valdettaro (2010). Fig. 1.4 shows the formation of internal shear layers and the velocity field with the length scale in terms of the Ekman number. At the known critical latitude for the spherical shell, marked as blue dots in Fig. 1.4, the thickness of the Ekman layer increases locally (Greenspan, 1990). At the critical latitude of 0°



(a) (b)

Figure 1.3: Stewartson layer in differential rotating spherical shell: a) cylindrical shear layer formation tangential to the inner sphere along Z, b) separated Ekman layers with their different widths (after Stewartson (1966)).

the Ekman layer is stationary in this equatorial region. In the oscillating mode 0 to 2Ω the critical latitude shift from the equatorial region up- and downwards, depending on the forcing frequency.

Ekman layer

Another characteristic in rotating fluids is the occurrence of Ekman layers. It's a boundary layer between geostrophic flow and its viscous (no-slip) closure. It shows also different layers in between, like the Stewartson layer, see Fig. 1.3 (b). For the velocities, no-slip boundary conditions hold within this boundary layer, that takes friction effects into account. The Ekman layer is present below the geostrophic wind and takes up the main part of the atmospheric boundary layer over 2000 meters. To form an Ekman layer the system has to rotate, so the Coriolis force ($N\vec{\Omega} \times \vec{u}$) changes the direction of the flow, proportionally to the magnitude of the velocity, also called Ekman spiral. Above this layer a laminar flow almost prevails, where pressure force and Coriolis force dominate the flow motion. Detailed description of the atmospheric layer, particular the Ekman layer is given in Cushman-Roisin (1994); Malberg (2007); Etling (2008); Klose (2008). For theoretical investigations on the Ekman layer e.g. see also Stewartson (1966); Hollerbach (1994); Kerswell (1995); Calkins et al. (2010).

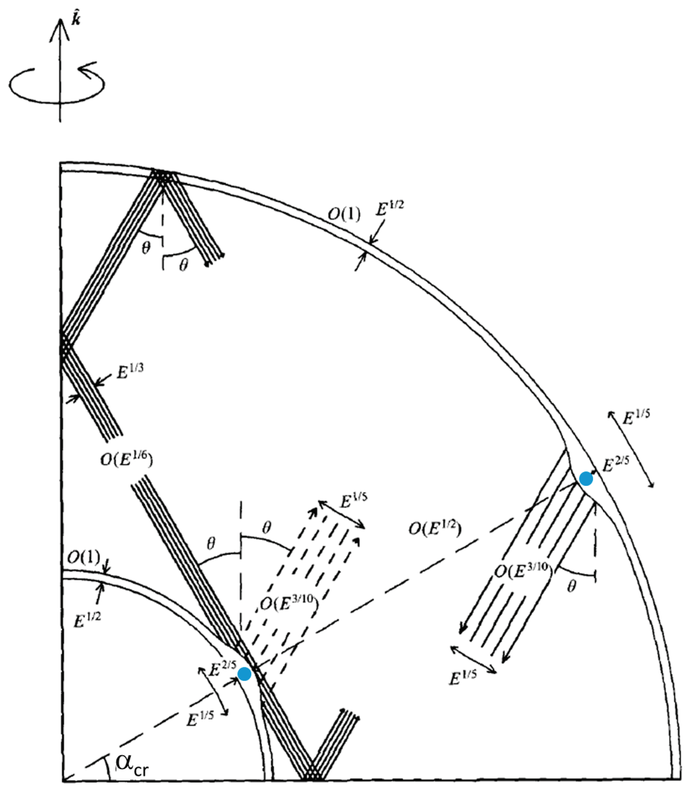


Figure 1.4: Velocity field and length scale in orders of the Ekman number of internal shear layers in a rotating spherical shell, where the background velocity field is $O(Ek^{1/2})$. The bulge at the inner and outer denotes the critical latitude (figure from Kerswell (1995)).