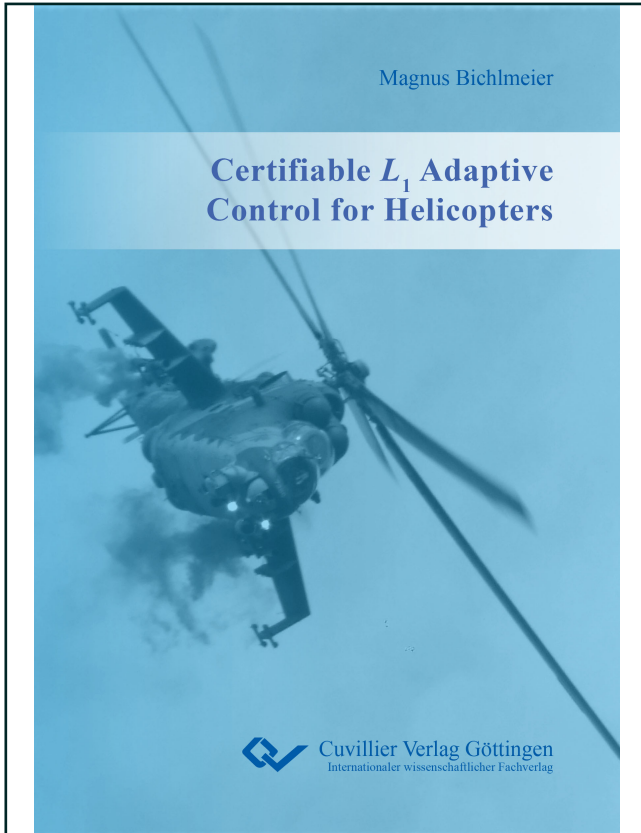




Magnus Bichlmeier (Autor)  
**Certifiable  $L_1$  Adaptive Control for Helicopters**



<https://cuvillier.de/de/shop/publications/7276>

Copyright:

Cuvillier Verlag, Inhaberin Annette Jentsch-Cuvillier, Nonnenstieg 8, 37075 Göttingen,  
Germany

Telefon: +49 (0)551 54724-0, E-Mail: [info@cuvillier.de](mailto:info@cuvillier.de), Website: <https://cuvillier.de>



# Contents

List of Figures	viii
Acronyms	xi
Notation and Symbols	xii
<b>1 Introduction</b>	<b>1</b>
1.1 Motivation . . . . .	1
1.2 Controller Requirements and Objectives . . . . .	3
1.3 Chapter Overview . . . . .	5
1.4 Contributions of this Thesis . . . . .	6
<b>2 Background</b>	<b>8</b>
2.1 Helicopter Dynamics . . . . .	8
2.2 System Description . . . . .	12
2.3 Offline System Identification . . . . .	14
2.4 The Baseline Controller . . . . .	16
2.5 Introduction to $\mathcal{L}_1$ -Control . . . . .	26
2.5.1 The Idea . . . . .	26
2.5.2 Explanation to the Piece-Wise Constant Adaptive Law . . . . .	32
2.5.3 Introduction to Output Feedback for Non-SPR Desired Dynamics . . . . .	34
2.5.4 Performance Effects of the Prediction Error: Issues and Solutions . . . . .	35
2.5.5 Performance and Robustness . . . . .	37
2.6 Augmenting and Standalone $\mathcal{L}_1$ -Controllers . . . . .	38
2.7 Internal Model Based Control . . . . .	42
<b>3 Design of the Input Channel to the Predictor</b>	<b>45</b>
3.1 Input Signal Merging . . . . .	45
3.2 Trim . . . . .	47
3.3 Decoupling of Cross-Couplings . . . . .	47
3.4 Structure . . . . .	48
3.5 Modeling of Response Lags . . . . .	51
<b>4 <math>\mathcal{L}_1</math>-Control for Pitch, Roll, Yaw</b>	<b>53</b>
4.1 In General . . . . .	53
4.2 State Feedback . . . . .	54



---

4.3	Output Feedback . . . . .	56
4.4	Outer Loops for the Standalone $\mathcal{L}_1$ -Controller . . . . .	58
<b>5</b>	<b><math>\mathcal{L}_1</math>-Control for Vertical Speed in Hover</b>	<b>61</b>
5.1	Background . . . . .	61
5.2	Controller Core . . . . .	62
5.3	Command Signal Processing . . . . .	64
5.4	Extensions and Enhancements . . . . .	66
5.5	Alternative Structure . . . . .	67
<b>6</b>	<b>Certification Strategy</b>	<b>68</b>
6.1	Introduction . . . . .	68
6.2	Core Reasoning . . . . .	69
6.3	Additional Supportive Criteria . . . . .	71
<b>7</b>	<b>Simulation Results</b>	<b>74</b>
7.1	Frequency Domain: Baseline Controller . . . . .	76
7.2	Frequency Domain: Augmentation with State Feedback . . . . .	78
7.3	Frequency Domain: Standalone $\mathcal{L}_1$ -Control in State Feedback . . . . .	80
7.3.1	Raw Adaptive Law . . . . .	80
7.3.2	Recursive Adaptive Law . . . . .	82
7.4	Frequency Domain: Standalone $\mathcal{L}_1$ -Control in Output Feedback . . . . .	84
7.5	Time Domain: Standalone $\mathcal{L}_1$ -Control in State Feedback – Raw Adaptive Law . . . . .	86
7.6	Vertical Speed Controller . . . . .	87
<b>8</b>	<b>Conclusion and Future Work</b>	<b>88</b>
8.1	Conclusion . . . . .	88
8.2	Future Work . . . . .	91
<b>A</b>	<b>Definitions</b>	<b>92</b>
A.1	General Definitions . . . . .	92
A.2	Coordinate Frames . . . . .	96
<b>B</b>	<b>Alternatives for Decoupling of Cross-Couplings in Augmentation</b>	<b>98</b>
<b>C</b>	<b><math>\mathcal{L}_\infty</math>-Stability</b>	<b>100</b>
C.1	Introduction and Conditions . . . . .	100
C.2	Proof of Theorem C.1.1 . . . . .	100
<b>D</b>	<b>Equivalent State Feedback Systems</b>	<b>102</b>
<b>E</b>	<b>Performance Bounds in State Feedback</b>	<b>104</b>
E.1	Definitions, Assumptions, Descriptions . . . . .	105
E.2	Reference System . . . . .	111
E.3	Core Proof . . . . .	112

E.4	Proof of Lemmas . . . . .	117
E.4.1	Proof of Lemma E.2.1 . . . . .	117
E.4.2	Proof of Lemma E.3.1 . . . . .	118
E.4.3	Proof of Lemma E.3.2 . . . . .	120
E.4.4	Proof of Lemma E.3.3 . . . . .	121
E.5	Alternative, Recursive Adaptive Law . . . . .	122
<b>F</b>	<b>Performance Bounds in Output Feedback for Non-SPR Desired Dynamics with a Recursive Adaptive Law</b>	<b>126</b>
F.1	Definitions, Assumptions, Descriptions . . . . .	127
F.2	Reference System . . . . .	136
F.3	Core Proof in Initial Broken-Loop Mode . . . . .	136
F.4	Core Proof in Standard Closed-Loop Mode . . . . .	148
F.5	Proof of Lemmas . . . . .	160
F.5.1	Proof of Lemma F.1.1 . . . . .	160
F.5.2	Proof of Lemma F.1.2 . . . . .	165
F.5.3	Proof of Lemma F.1.3 . . . . .	165
F.5.4	Proof of Lemma F.2.1 . . . . .	165
F.5.5	Proof of Lemma F.3.1 . . . . .	167
F.5.6	Proof of Lemma F.3.2 . . . . .	168
F.5.7	Proof of Lemma F.4.1 . . . . .	168
F.5.8	Proof of Lemma F.4.2 and Lemma F.3.3 . . . . .	170
F.5.9	Proof of Lemma F.4.3 . . . . .	172
F.5.10	Proof of Lemma F.5.1 . . . . .	173
F.5.11	Proof of Lemma F.5.2 . . . . .	175
<b>G</b>	<b>Signal Hedging with Saturation</b>	<b>176</b>
G.1	Introduction and Conditions . . . . .	176
G.2	Proof of Theorem G.1.1 . . . . .	177
<b>H</b>	<b>The Effect of <math>a_{SP}</math> on <math> \tilde{x} </math>, <math> x_{ref} - x </math>, <math> x_{ref} - \hat{x} </math></b>	<b>179</b>
H.1	Description . . . . .	179
H.2	Example . . . . .	180
H.3	Explanation . . . . .	182
<b>I</b>	<b>Controller Robustness against Time Step Variations</b>	<b>187</b>
I.1	Description . . . . .	187
I.2	Example . . . . .	188
I.3	Explanation . . . . .	189
<b>J</b>	<b>Controller Robustness against Measurement Noise</b>	<b>191</b>
J.1	Description . . . . .	191
J.2	Example . . . . .	193
J.3	Proof of Theorem J.1.1 . . . . .	196
J.4	Proof of Theorem J.1.2 . . . . .	197
J.5	Proof of Theorem J.1.3 . . . . .	197



---

<b>K</b>	<b>Modeling of Inertia Effects with a Gyroscopic Term</b>	<b>199</b>
<b>L</b>	<b>Alternative Predictor in an Error Space</b>	<b>201</b>
<b>M</b>	<b>Alternative State Predictor Enclosing Baseline Controller States</b>	<b>203</b>
	M.1 Definitions . . . . .	203
	M.2 Core Statements . . . . .	204
	M.3 Necessity of the Alternative Predictor . . . . .	207
	M.4 Proof of Theorem M.3.1 . . . . .	207
	M.5 Example . . . . .	209
<b>N</b>	<b>Examples of Practical Verification and Falsification</b>	<b>212</b>
<b>O</b>	<b>Simulation Setup</b>	<b>215</b>
	O.1 Concept . . . . .	215
	O.2 Structure . . . . .	216
	O.3 Parameter Scheduling in State Feedback . . . . .	218
	O.4 Actuators . . . . .	218
	O.5 Noise Modeling . . . . .	220
	O.6 Miscellaneous . . . . .	220
	<b>Bibliography</b>	<b>222</b>