1 Introduction

1.1 Problem Definition and Objectives of this Thesis

The main problem in portfolio management is the question how an investor should distribute his wealth to gain the best possible expected return for the amount of risk he is willing to suffer from. To solve this problem, the groundbreaking work of Markowitz (1952) serves as the theoretical basis on which several capital market models were developed.

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) is the most established of these models. Although it has suffered from many critical remarks throughout the years, it remains a standard model until today. Besides its extensive presence in theoretical research, it is also widely employed in practice. This is due to the main advantage of the CAPM compared to subsequently developed and more sophisticated models: its convenient applicability combined with comparatively little data requirements. The most promising successor of the CAPM is the Fama-French three-factor model, which was introduced by Fama & French (1993). This model is already widespread in research, however, it is only rarely used in practice. Consequently, the CAPM remains the dominant model in practical applications and it does not seem to lose this position in the near future.

The essential factor of a stock in the equilibrium equation of the CAPM is the beta factor, which states the systematic risk contribution of this stock to the overall risk of a portfolio. The beta factor in particular can for example be used to assess listed companies, calculate market oriented capital costs or optimize portfolio structure without estimating the whole variance-covariance-matrix.[1] For all these applications the accuracy of the beta factor is of special interest. Because the market portfolio is not observable and the 'true' beta is unknown, knowledge about deviations between beta factors due to different input parameters for their estimation gets essential. This is also true for the Fama-French model as it is an extension of the CAPM and the beta factor remains an important parameter. However, the Fama-French model additionally considers two further factors related to size and book-to-market influencing the return of a stock.

Looking at the input parameters the model requires, one important input parameter in both models is the interval length of the underlying return data. Although weekly data are commonly used to calculate betas in practice, only little has been said about the variation of weekly beta

[1] Examples of other applications can be found in Bruner et al. (1998) and Elton et al. (2007), pp. 131f.

factors estimated on the basis of different days of the week. This is particularly surprising because data vendors often employ one single weekday as a week's representative for estimating weekly betas. Bloomberg, for example, utilizes Friday betas as default calibration for calculating calendar weekly betas and Bloomberg is a source that practitioners often use. Of course, this praxis is in line with intuition as the weighted sum of all stock betas in an index applied as a market proxy – independent of the weekday chosen as base date – must correspond to one. For this reason the change of base date leading to an increase in the weekly beta of one company must result in a decrease in the weekly beta of (at least) another company to maintain the value of the sum. Why this phenomenon should exist is not intuitively plausible.

Nevertheless, the literature shows that weekly betas may deviate due to estimation on the basis of different weekdays and this is an important element of uncertainty. Therefore, one must be aware that the choice of weekday possibly is an influential parameter in estimating weekly betas. For a well-informed choice of base date especially the type of differences between betas estimated on the basis of different weekdays is important. The characteristics of these differences in betas, however, have not been empirically analyzed so far.

Consequently, relevant questions remain unanswered and reveal a gap, which the present thesis closes. Thereby, the following questions are focused:

- Do weekly betas deviate due to estimation on the basis of different weekdays?
- Do stable size relations between betas estimated on the basis of different weekdays exist?
- Does the answer to the questions above depend on the length of the timeline for beta estimation or on the considered national market?

To answer these questions, returns of large stocks within a broad and representative long-term empirical data set are used in this thesis. The analyses also use OLS and three alternative estimation methods for robustness purposes. To additionally examine if the results found are stable with regard to different frameworks, the model context is changed. It is switched from the standard single-index model (CAPM) to the most popular multi-index model (Fama-French three-factor model). First, the questions presented above are examined separately in both contexts, afterwards the results are compared to each other.

As mentioned above, this thesis examines the forecited research questions through empirical analysis. In the beginning, the theoretical framework and the mathematical basics for the empirical analysis are presented. Based on the methodological groundwork, empirical analyses are established in order to analyze the research questions. The first empirical analysis deals with the influence of weekday choice for betas estimated in the CAPM. The second empirical analysis investigates betas and the two additional factors estimated in the Fama-French three-factor model. Again, the influence of the weekday chosen as base date for factor estimation is focused.

1.2 Course of Investigation

In order to analyze the research questions mentioned in the former subsection, the course of investigation is as follows. Chapter 2 gives a short overview of the model framework used in this thesis. First, the Capital Asset Pricing Model (CAPM) is explained in Section 2.1, while its importance and usage as a standard model throughout the world is exposed in Section 2.2. As the beta factor takes a prominent place, in the CAPM as well as in the investigation of this thesis, this parameter is explained thoroughly in Section 2.3. Finally, the Fama-French three-factor model is presented in Section 2.4.

In contrast to the theoretical framework explained in Chapter 2, Chapter 3 deals with the mathematical methods used in the course of the analysis, i.e. with aspects regarding the estimation of beta factors. In this regard, the application of the market model is explained in Section 3.1, while the methodological standard approach for estimating beta factors, Ordinary Least Squares (OLS) regression is introduced in Section 3.2. Subsequently, Section 3.3 turns to assumptions necessary for applicability of OLS. After giving an overview of the standard method, Section 3.4 turns to alternative regression methods which are robust against violation of the above mentioned assumptions. In addition to the choice of regression method, the input parameter specification is another possibility to influence the resulting beta factors. Section 3.5 turns to the possible variants regarding returns and return intervals, market proxies or length of timeline including their influence on the resulting beta factors.

The empirical analysis of Chapter 4 aims at clarifying if a weekday effect in beta factors calculated on the basis of different weekdays is present when using the CAPM. The fundamentals of this empirical analysis are explained in Section 4.1 followed by a literature review in Section 4.2. Section 4.3 introduces the empirical data set and Section 4.4 explains the estimation of beta factors. These beta factors are characterized in Section 4.5 before the empirical methodology is explained in Section 4.6 with the use of several aspects already explained in Chapter 3. The empirical results are exhibited in Section 4.7. The main results of this Chapter are finally subsumed in Section 4.8.

The empirical analysis of Chapter 5 exceeds the previous analysis by using the Fama-French three-factor model and additionally comparing the results for both models. Hence this Chapter aims at analyzing if the results for weekday betas in the CAPM context also hold within another model context, i.e. the Fama-French model. The relevant research questions are formulated in Section 5.1, while Section 5.2 gives a short literature review. The data set is characterized in Section 5.3, followed by an explanation of factor estimation in Section 5.4 and the characterization of these factors in Section 5.5. Section 5.6 describes the empirical methodology adapted to the new model context. Afterwards, the empirical results are presented in Section 5.7 before the main results of this Chapter are subsumed in Section 5.8. In each Section the comparison between Fama-French model and CAPM follows in the last part of the Section, after the consideration of the Fama-French model is completed. Chapter 6 finally concludes.

2 Capital Market Models

In this Chapter the capital market models used in the analyses are shortly introduced. Section 2.1 starts with a general introduction of the CAPM, while Section 2.2 focuses on both, empirical critic and importance of the model. Afterwards, in Section 2.3 the beta factor as the key factor in the CAPM is introduced in more detail. Finally, the Fama-French three-factor model is explained in Section 2.4.

2.1 The Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM), which is a standard model in finance, was independently developed by Sharpe (1964), Lintner (1965) and Mossin (1966). It shows the risk-return relationship of an asset in equilibrium. The CAPM is based on the portfolio selection theory of Harry Markowitz (1952, 1959), that was 1990 rewarded with the Nobel Prize. His portfolio selection theory assumes that investors assess a portfolio of stocks by considering solely expected portfolio return μ_P and portfolio risk in the form of return standard deviation σ_P ($\mu - \sigma$-principle).[2] Based on this fundamental assumption about investor behavior, the CAPM – like every theoretical model – reduces complexity by several further assumptions.

First, transaction costs and personal income tax do not exist. Second, assets are infinitely divisible, so that every investor is able to participate in every stock, regardless of his wealth. Third, individual investors cannot affect stock prices by their actions. Due to this assumption, investors in total determine prices by buying or selling activity. Fourth, investors base their investment decisions only on expected return μ and standard deviation σ of portfolio returns. Fifth, short sales are allowed and their amount is not restricted. Sixth, unlimited lending and borrowing at the riskless rate is possible. Seventh, all investors consider mean μ and standard deviation σ of returns in exactly the same single period in time. Eighth, all investors have the same expectations regarding expected returns, the variance of returns and the correlation matrix of portfolios. The correlation matrix is important, as it displays correlations between returns of every pair of stocks in a portfolio. Ninth, all assets, including human capital, are marketable.[3] Under these assumptions it can be shown that in the equilibrium the market portfolio is the risky

[2] For an extensive presentation of portfolio selection theory see for example Elton et al. (2007), pp. 41ff., and Breuer et al. (2010), pp. 137ff. As Markowitz considers only fully rational investors, irrational behavior is not part of the theory. This deviating behavior is analyzed in behavioral finance, see for example Shleifer (2003).

[3] See Elton et al. (2007), p. 284f.

portfolio any investor will hold. Individual differences in risk aversion simply lead to different combinations of the market portfolio with riskless borrowing or lending.[4] This knowledge leads to the insight that the market portfolio and the return of the riskless asset play an important role in the CAPM equation. In fact, the equilibrium relationship for any security, the so-called security market line (SML), is given by

$$\mu_P = r_f + \beta_P \left(\mu_M - r_f \right), \text{ with } \beta_P = \frac{\sigma_{PM}}{\sigma_M^2}. \tag{2.1.1}$$

This is the standard equation of the CAPM[5] and depicts the equilibrium return on all portfolios as well as all securities. Here, μ_P is the expected return of a portfolio or security, r_f is the return of the riskless asset, μ_M is the expected return of the market portfolio, σ_M^2 is the variance of the market return, σ_{PM} is the covariance between the returns of the considered portfolio or security and the market portfolio and β_P is the beta factor.

The equation of the CAPM states that the price of time (represented by the return of the riskless asset, r_f) added to the product of the market price of risk ($\mu_M - r_f$) and the contribution of the security to the market risk (β) results in the equilibrium expected return on any security. This return is a linearly increasing function of the systematic risk β, which can be clearly seen in (2.1.1). In addition, only systematic risk[6] influences the equilibrium return of a security while diversifiable risk is not considered.[7]

2.2 Empirical Evidence and Relevance of the CAPM

Although the CAPM is a standard model in finance, empirical evidence regarding its validity is ambivalent.[8] The central critique of the CAPM was formulated by Roll (1977): „The theory is not testable unless the exact composition of the true market portfolio is known and used in the tests. This implies that the theory is not testable unless all individual assets are included in

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[4] This leads to the two mutual fund theorem. This theorem declares that by combining the riskless asset and a market fund, every investor is able to construct an optimal portfolio.

[5] For derivation and detailed description of the CAPM see for example Elton et al. (2007), pp. 286ff.; Copeland et al. (2011), pp. 149ff.; Sharpe et al. (1999), pp. 227ff.; Bodie et al. (2011), pp. 280ff. or Brown & Reilly (1997), pp. 287ff.

[6] Systematic risk, also called nondiversifiable risk, is the amount of risk which is related to marketwide risk drivers and cannot be eliminated by diversification. In contrast, firm-specific risk, which is also called nonsystematic or diversifiable risk, can be eliminated by diversification. See Bodie et al. (2011), p. 197.

[7] See Elton et al. (2007), p. 296ff. For real-world applications beta is usually estimated by a historical time series regression (see Section 3.1).

[8] For an overview over numerous empirical studies regarding validity of the CAPM see for example Jensen (1972); Campbell et al. (1997), pp. 211f., or Fama & French (2004), p. 25ff.

the sample."[9] For this reason, one part of the CAPM-critical literature tries to improve tests of the CAPM. This is done either by improved econometric methods[10] or by using better proxies of the unknown parameters, especially the expected returns in the CAPM. As the CAPM is based on expected returns while tests of the CAPM typically use realized returns[11], this discrepancy is a problem.

Another part of literature takes the empirical weakness of the CAPM as reason for developing alternative capital market models. In this case data-oriented models are very popular to explain observed average returns. Typical representatives of this branch are factor models, mostly introducing macroeconomic or firm-specific factors into the model. The most popular data-driven model is the Fama-French three-factor model[12], but there are also others[13]. Furthermore, several extensions of the standard CAPM were developed to gain more realistic model variants with less restrictive model assumptions.[14] In this context the intertemporal CAPM (ICAMP) by Merton (1973), the consumption oriented CAPM (CCAPM) by Breeden (1979),[15] the CAPM with imperfect information by Merton (1987), the Tax-CAPM by Brennan (1970) and Jonas et al. (2004) as well as the Liquidity CAPM by Kempf (1999), Jacoby et al. (2000) and Acharya & Pedersen (2005) have to be mentioned. Nevertheless, these model variants never replaced the standard CAPM and still do not have great impact in theoretical research or practical business live. Solely the Fama-French three-factor model reached popularity and broad acceptance in research.

In asset management, performance measurement of stock portfolios is performed with CAPM-oriented measures such as Jensen's alpha, which is calculated as the difference between the average portfolio return and its risk adjusted model return of the CAPM.[16] In addition to performance evaluation in research[17] Jensen's alpha is also used by data vendors (e.g. Morningstar)[18]. Regarding the calculation of capital costs, the IDW (Institut der Wirtschaftsprüfer)[19]

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[9] See Roll (1977), p. 130. The critique in this article is generally known as "Roll's critique".
[10] Examples for advanced test methods are the contributions of Jagannathan & Wang (1996) and Lewellen & Nagel (2006) regarding the conditioned CAPM, where beta and the market risk premium are allowed to vary over time.
[11] For an overview of papers not using realized returns see Hagemeister (2010), pp. 137f.
[12] For a more detailed consideration of the Fama-French three-factor model see Section 2.4.
[13] Chen et al. (1986), Flannery & Protopapadakis (2002) and Shanken & Weinstein (2006) use macroeconomic factors, Fama & French (1992), Chan et al. (1998) and Brennan et al. (2005) use firm-specific factors.
[14] See Hagemeister (2010), pp. 136ff.
[15] For a detailed presentation of ICAPM and CCAPM see for example Schneider (2001).
[16] See Jensen (1968), pp. 389ff.
[17] E.g. Evans (2010), Fama & French (2010).
[18] See Brückner (2013), p. 68.
[19] The IDW is a registered association of auditors and auditing firms in Germany, whose members represent more than 80% of all German auditors. See IDW (2014).

has recommended the use of CAPM since 2000[20] and the OLG (Regional Appeal Court) in Düsseldorf stated in a decision from May 27th 2009 that for valuation purposes the CAPM is the most important model for estimating capital costs (I-26W 5/07). Similarly, regulatory practices show a preference for the CAPM in calculating costs of equity capital in several countries.[21] Likewise, literature presents the use of CAPM for estimation of capital costs as a standard approach[22] and empirical studies surveying common practice in capital cost calculation find a large majority of firms actually using the CAPM for this purpose.[23]

2.3 The Beta Factor in the CAPM

The essential factor of a stock in the equation of the CAPM is the beta factor. It states the systematic risk contribution of this stock to the overall risk of a portfolio and has therefore a strong impact on the expected return of this stock. While stocks perfectly depicting the market return obviously have a beta factor of 1, betas of more volatile stocks (so-called aggressive stocks) are larger than 1 and betas of stocks that are less volatile than the market (so-called defensive stocks) are smaller than 1.[24]

The beta factor in particular has several applications in practice. It is used to asses listed companies from an investor's perspective[25] or calculate market oriented capital costs[26]. As many investors are not able to estimate beta factors due to lack of data, they use betas estimated by data provider such as Bloomberg, Value Line and Standard & Poors (S&P)[27] or so-called Beta Books[28]. Similarly, companies not quoted on the stock exchange are assessed using an industry-beta as an approximation for the unknown company-beta[29]. In portfolio management applica-

[20] See IDW (2000).
[21] See Sudarsanam et al. (2011), p. 26.
[22] E.g. Koller et al. (1996), p. 39, Pratt & Grabowski (2010), pp. 103ff.
[23] E.g. Bruner et al. (1998), p. 17 (81% of respondents), Graham & Harvey (2001), p. 7 (73% of respondents).
[24] See Sharpe et al. (1999), p. 183f.
[25] E.g. Deutsche Börse Group calculates and publishes beta factors on their homepage as an investor service; similarly beta factors are published as quantitative measures for stocks in the 'Börsen-Zeitung'.
[26] E.g. Bruner et al. (1998) p. 17; for an extensive discussion of various forms and applications of beta in estimating capital costs see Pratt & Grabowski (2010), pp. 159ff.
[27] See Bruner et al. (1998), p. 20.
[28] For the German capital market Dörschel et al. (2010) published such an overview of beta factors and related facts, Morningstar even distinguishes between a "Beta Book" for individual companies and a "Cost of Capital Yearbook" for industry betas, see Pratt & Grabowski (2010), p. 727f.
[29] See Bowman & Bush (2006); industry-betas are also available by the before mentioned data sources.

tions the beta factor enables portfolio structure optimization without estimating the whole co-variance-matrix[30] and plays a part in performance measurement[31]. In some cases there are even portfolio strategies, aiming to reach a special portfolio beta. Giving one example, index tracking aims at reaching a portfolio beta of 1.[32]

In addition to practical applications beta factors are also intensely, albeit controversially, discussed in capital market research. This can especially be seen by active discussions over decades of years regarding the issue if beta is a useful parameter at all.[33] For this question all variable parameters in the estimation of beta (market portfolio proxy, return interval length, regression length and return calculation) have a strong influence on the resulting conclusion. Therefore they are analyzed in detail.[34] Likewise, reasons for specific behavior of betas[35], their stability over time[36] and lots of other questions are of interest and an active area of research. However, despite all controversial discussions beta remains an important and widely used parameter in capital market research and practice, and still there is no promising successor in sight.

2.4 The Fama French Model

As mentioned above the CAPM came under criticism due to testability issues and ambivalent empirical evidence. Empirical studies showed other factors which, in addition to the market return, may also influence expected stock returns. These influences are often called anomalies, as the CAPM is not able to explain them.[37] The first anomaly is related to company size and therefore called size-effect. Banz (1981) shows significant higher returns for companies with small market capitalization than for companies with large market capitalization. Roll (1981) attributes the effect back to illiquidity. However, Reinganum (1982) shows that illiquidity only

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[30] See Sharpe et al. (1999), pp. 198f. Using Markowitz portfolio optimization the covariances for each pair of risky assets need to be estimated. For n assets this results in $(n^2 - n)/2$ covariances. By using the market model approach underlying the CAPM the covariances needed can be expressed by the relation of every stock to the market index. Therefore for n assets only n beta factors are needed to gain the covariance relationships. Considering all needed variables the number reduces from $(n^2 - n)/2$ for Markowitz portfolio optimization to $3n + 2$ when using the market model. In addition the use of beta factors allows applying simplified methods for optimizing portfolio structure, when short selling is not possible. See Elton et al. (1976), pp. 1343ff.

[31] The reward to volatility ratio (Treynor ratio) $T_P = \frac{\mu_P - r_f}{\beta_P}$ measures the expected excess return of the portfolio standardized with the beta factor of the portfolio. See Treynor (1965).

[32] Regarding problems of index tracking see for example Roll (1992).

[33] E.g. Grinold (1993); Clare et al. (1997); Isakov (1999); Hsia et al. (2000).

[34] For a more detailed consideration of parameter variability and related literature for estimating beta factors see Section 3.5.

[35] E.g. Patton & Verardo (2012).

[36] E.g. Alexander & Chervany (1980).

[37] See Fama & French (1996).

partly explains the effect and still finds a significant size-effect. Currently, transaction costs and liquidity risks are possible candidates to explain the size-effect.[38]

The second anomaly is related to book-to-market ratio and therefore called value-effect[39]. Rosenberg et al. (1985) show a positive correlation between average returns and book-to-market ratio in the USA, Chan et al. (1991) show the same effect on the Japanese market. To account for the value-effect, Lakonishok et al. (1994) assume overreactions of market participants.[40]

Based on evidence on these two empirical anomalies Fama & French (1992) confirm the existence of size- as well as value-effect for the US market. Fama & French (1993) incorporate both effects in an extension of the CAPM, to explain capital market returns better than the CAPM is able to. The resulting three-factor model[41] is

$$\mu = r_f + \beta_{FF} \cdot (\mu_M - r_f) + smb \cdot SMB + hml \cdot HML \tag{2.4.1}$$

where μ is the expected return of a Fama-French portfolio[42], r_f is the return of the riskless asset, β_{FF} is the regression coefficient of the expected excess return of the market portfolio over the riskless rate $(r_M - r_f)$, smb is the regression coefficient of the SMB-portfolio return ("Small Minus Big", a size factor designed to mimic the size effect in returns) and hml is the regression coefficient of the HML-portfolio return ("High Minus Low", a value-factor designed to mimic the value effect in returns). The regression coefficients determine the contribution of the excess market return, the SMB- and HML-return in explaining stock and bond returns.[43]

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[38] See van Dijk (2011) for an overview.

[39] Often stocks with high book-to-market ratios or low price-earnings respectively price-cashflow ratios are called value stocks. These are contrasted to stocks with contrary ratios or high growth in sales respectively high growth in profits, which are called glamour- or growth-stocks. See for example Fama & French (1996).

[40] The authors suppose that market participants overestimate growth opportunities of stocks with high book-to-market ratios and underestimate stocks of distressed companies.

[41] Fama & French (1993) identify five common risk factors for explaining the returns on stocks and bonds. However, the three-factor model does equally well in explaining common time-series variation in stock returns. See Fama & French (1993), p. 54.

[42] Fama & French (1993) used the returns of 25 portfolios as μ when developing their model. The advantage of using portfolio returns instead of stock returns for fitting an empirical model is that portfolio returns are more smooth and therefore more suitable for fitting a generally valid model. The portfolios are constructed each year at the end of June ad are used from July to the end of June of the following year. First, 5 portfolios based on size (more specifically, based on quintiles of the ME values on the NYSE) and 5 portfolios based on BE/ME-ratio (BE/ME quintiles on the NYSE, where the BE/ME for June is BE for the last fiscal year end divided by ME for December of the last year) are formed. The 25 portfolios are the intersections of the both portfolio groups explained above. See Fama & French (1993).

[43] The Fama-French three-factor model is constructed based on empirical observations. Unless in case of the CAPM, there is no theoretical model behind the equation. For this reason it is not possible to express β_{FF}, smb and hml by an equation other than the regression estimator for their calculation as it was possible for the CAPM-β in Section 2.1.

The three factors in the Fama-French model are empirically motivated, which is the main difference to the theoretically founded CAPM. Therefore the specifications – especially regarding the SMB- and HML-parameter – are arbitrary to a certain degree.

Fama and French construct 2 portfolios based on Market equity (ME)[44] to consider the size of stocks. They calculate the median of ME and sort all stocks with ME values higher than the median in the "Big" portfolio. All remaining stocks are sorted into the "Small" portfolio. Likewise, all stocks are sorted into 3 portfolios based on their ratio of book equity to market equity (BE/ME)[45] to account for the value effect. Here the splits are made at the 70th BE/ME percentile and the 30th BE/ME percentile. The portfolio including the largest BE/ME ratios is named "Value", the middle portfolio is called "Neutral" and "Growth" includes stocks with the lowest BE/ME ratios. Stocks with negative BE are excluded.

Each year at the end of June, 6 portfolios are constructed by the intersections of the 2 size and 3 value portfolios presented above.[46] Their names and characteristics are summed up in Figure 2.1.

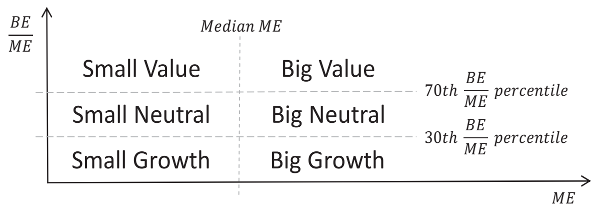

Figure 2.1: 6 Fama-French portfolios formed on size and book-to-market

Using these 6 portfolios the returns of SMB and HML are generated. SMB is the average return on all three "Small" portfolios minus the average return on all three "Big" portfolios, i.e.

$$\begin{aligned} SMB = &\frac{1}{3}(SmallValue + SmallNeutral + SmallGrowth) \\ &-\frac{1}{3}(BigValue + BigNeutral + BigGrowth). \end{aligned} \tag{2.4.2}$$

44 Fama & French (1993) use NYSE market equity values at the end of June of each year.
45 Likewise, Fama & French (1993) use NYSE BE/ME ratios. The ratios are calculated on the end of June based on the BE value for the last fiscal year end divided by ME for December of the last year.
46 The portfolios for July of one year to June of the next year include all stocks of NYSE, AMEX and NASDAQ for which ME data are available (for December of the last year and June of the actual year), as well as positive BE data (for the next year). All data are based on CRSP database.

Similarly, HML is the average return on both "Value" portfolios minus the average return on both "Growth" portfolios, i.e.

$$HML = \frac{1}{2}(SmallValue + BigValue) - \frac{1}{2}(SmallGrowth + BigGrowth)\,. \qquad (2.4.3)$$

The market parameter in the equation is the excess market return $(r_M - r_f)$. r_M is the return on all value weighted stocks including stocks with negative BE. r_f is the one-month bill rate.[47]

Fama and French initially constructed the three-factor model for the US market and analyzed R^2 values of 25 stock portfolios sorted by size and BE/ME ratios. They find that their model explains 83% – 97% of stock portfolio return variation while the CAPM – that is the market factor alone – only explains 61% – 92% of return variation for the above mentioned 25 portfolios.[48] The model is also popular in research as it is robust against further anomalies.[49] Only the short-term momentum effect[50] found by Jegadeesh & Titman (1993) and Rouwenhorst (1998) cannot be explained.

Although the model of Carhart (1997) not only includes size and value effects like the Fama-French three factor model, but also the momentum effect, the model is rather unknown. It is neither object of active academic research, nor widely known or applied in practice. Possible reasons are data availability and the data set on which Carhart developed his model. While Fama-French publish their results and the data they based their analysis on, the data set underlying Carhart's model is not available. This makes it much more difficult for researchers to execute continuative research and extend, adapt or otherwise use the model. In addition, Fama-French used stock portfolios for fitting their model, Carhart applied mutual fund data for fitting his. Although the result based on mutual funds is not mandatorily transferable to stocks as the more general group, successive studies transferring the results of Carhart's model to stocks are very rare.[51]

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[47] See Fama & French (1993).
[48] See Fama & French (1993). There is no explicit relationship between size or BE/ME ratio and the percentage of explained stock return variation. Several years later the authors transferred the three-factor model to four regions of international stock markets, see Fama & French (2012).
[49] See Lakonishok et al. (1994) and Fama & French (1996).
[50] The observed phenomenon that stocks which performed well in the past tend to perform well in the near future and stocks which performed badly in the past tend to perform badly in the near future is called momentum effect. To account for the momentum effect Carhart (1997) extended the Fama-French three-factor model to a four-factor model by including an additional one-year momentum factor.
[51] See L'Her et al. (2004).

Furthermore, the fact that the Fama-French three-factor model improves the explanation of variation in stock returns compared to the CAPM has been found several times.[52] For Carhart's model, however, Hanauer et al. (2013) and Liew & Vassalou (2000) find that the benefit of an additional fourth factor is only marginal. Similarly, the extension of the Fama-French model by two additional bond market factors does not explain stock returns noticeably better.[53] In conclusion, the empirical studies in this thesis are executed for the CAPM as the standard model and the Fama-French three-factor model as a thoroughly evaluated model and the most promising successor of the CAPM. The importance and dispersion of the Fama-French three-factor model in research can be derived from the fact that researchers transferred the original model from the US to other capital markets. Local models exist for Canada[54], UK[55] and Germany[56]. Analogous to the USA the three-factor model explains the variation in stock returns in Canada, UK and Germany better than the CAPM does.[57]

As mentioned in Section 2.2, the three-factor model so far has not been able to oust the CAPM from its prominent position in practical financial applications although it is quite popular in research.[58] This is probably due to the dominant presence of the CAPM over decades and the appealing simplicity of its application. In addition, the more complicated three-factor model is not automatically better than the CAPM. Fama & French (1997) themselves find the CAPM and the three-factor model are equally imprecise in calculating industry cost of equity.

[52] See for example Fama & French (1993), Hanauer et al. (2013), Hussain et al. (2002) and Ziegler et al. (2007).
[53] See Fama & French (1993), Berkowitz & Qiu (2011) and Ziegler et al. (2007).
[54] See Berkowitz & Qiu (2001).
[55] See Hussain et al. (2002).
[56] See Hanauer et al. (2013), Artmann et al. (2012) and Ziegler et al. (2003).
[57] See Ziegler et al. (2003).
[58] In a survey of Graham & Harvey (2001), about 73% of 392 Chief Financial Officers of US corporations declare to use the CAPM to calculate the cost of equity capital. Multifactor-models are used only by a minor percentage. Similarly, 56% of 138 corporations of the CDAX use the CAPM to calculate the cost of equity capital in 2009 while not a single one uses the Fama-French model; see Homburg et al. (2011).

3 Methodological Background

Chapter 3 attends to the methodology used in the empirical studies in Chapters 4 and 5. After a short explanation of the relationship between the CAPM and the market model in Section 3.1, Section 3.2 and 3.3 focus on Ordinary Least Squares regression and its assumptions. Section 3.4 introduces alternative, robust regression methods and Section 3.5 gives an overview over the input parameters in empirical capital market models.

3.1 CAPM and the Market Model

Typically, the CAPM-beta is estimated with on a historical time series regression. This regression is based on the so-called market model[59], which is a special case of a single-index model[60]. There are two forms of the market model, here shown for the ith asset on the tth observation. In the standard form

$$r_{i,t} = \alpha_i + \beta_i r_{M,t} + \varepsilon_{i,t} \tag{3.1.1}$$

the index is represented by the return of the market portfolio[61], $r_{M,t}$. In the excess return form

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{M,t} - r_{f,t}) + \varepsilon_{i,t} \tag{3.1.2}$$

the index is represented by the excess return of the market portfolio compared to the riskless return, $r_{M,t} - r_{f,t}$.[62]

Both forms show a linear relationship between the asset return $r_{i,t}$ (excess return $r_{i,t} - r_{f,t}$) and the market return $r_{M,t}$ (market excess return $r_{M,t} - r_{f,t}$). The parameter α_i represents a market-independent, firm specific component of the asset return whereas β_i represents the sensitivity of the asset to changes in the market return. Graphically, α_i is the y-axis intercept and β_i is the

[59] The term market model was introduced by Fama (1968), p. 37.
[60] Single-index models are mainly characterized by their property to attribute all correlations of single assets to their correlation with one index. For an extensive presentation of single- and multi-index models see for example Elton et al. (2007), Chapters 7 and 8.
[61] As the market portfolio is not directly observable, a proxy needs to be chosen to substitute the market return (for a detailed discussion see Section 3.5.4).
[62] The standard form of the market model has the advantage that no riskless asset is needed, although the results are nearly the same as in the excess return model. See Black et al. (1972) or Hill & Stone (1980) or Elton et al. (2007), pp. 139ff.

slope of the regression line in a $r_M - r_i$ diagram. The residual $\varepsilon_{i,t}$ is an asset specific, risky return component which is uncorrelated to the market return.[63]

In addition to the CAPM assumptions, the market model is only valid when two additional assumptions hold: The joint distribution of asset and market returns as well as the parameters α and β of the considered asset are constant[64] over time.[65] As the CAPM makes statements about expected returns it must be converted into a form where historical data can be used. Only if this converted form is applied, the CAPM-beta can be estimated with a market model regression based on a historical time series. This means, that on average the realized returns must equal the expected rate of return, which is true when the following assumptions hold:

- In each period the return-producing process of the capital market is the market model in standard or excess return form.
- The expectations of the investors are rational, which means that the expectations of investors are analog to the predictions of the CAPM as the relevant economic theory.[66]

As the CAPM is an equilibrium model, the ideal capital market would always be in an equilibrium state and the CAPM holds in every period. In practice this is not probable, however, it is sufficient when at least no systematic deviations of capital market and CAPM assumptions exist to keep the validity of the CAPM plausible. As a result, the CAPM may be used in an empirical context whenever no systematic deviations between CAPM and the empirical observations on the capital market are found.

In contrast to the market model, α is not part of the CAPM's security market line. Therefore, market model and CAPM are only compatible for special values of α[67]. For the standard form, market model and CAPM are equal if $\alpha = (1 - \beta) r_f$[68]. For the excess return form, market model and CAPM are equal if $\alpha = 0$.[69]

When using the standard form of the market model, another assumption becomes relevant: the riskless asset return must be constant over time. If r_f is not constant, the standard market model

[63] See Kleeberg & Rehkugler (1998), pp. 438f.
[64] Constant parameters are not self-evident on real capital markets. Several empirical studies show that beta factors are not necessarily constant over time, but tend to revert to a long-term mean value (for example Zhang (2005), Petkova & Zhang (2005)). Consequently, a broad field in research is engaged in the stability of beta over time.
[65] See Winkelmann (1984), p. 17.
[66] See Huang & Litzenberger (1988), pp. 303f.; Elton et al. (2007), p. 335; Copeland et al. (2011), pp. 164f.
[67] For a derivation of the values of α_i for which market model and CAPM are compatible see Appendix 3.6.1.
[68] See Roll (1969), p. 275, Sharpe (1991), p. 497.
[69] See Merton (1973) or Ruppert (2010), pp. 435f. If $\alpha \neq 0$, the considered asset is mispriced or the market is not in an equilibrium state.